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DISTRACTIONS IN NON-VERBAL MATHEMATICAL PROBLEMS: SOME EFFECTS ON THE PROBLEM-SOLVING BEHAVIOR AND PERFORMANCE OF YOUNG CHILDREN

by

© JOHN PETER BANA

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Distractions in Non-Verbal Mathematical Problems: Some Effects on the Problem-Solving Behavior and Performance of Young Children submitted by John Peter Bana in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

A number of studies have shown that the inclusion of noise or distractions in word problems tends to increase problem difficulty. Piagetian and related research has revealed that young children experience difficulties in developing particular concepts due to their attention to irrevelant perceptual cues. There is some evidence that young children are also distracted in non-verbal mathematical problems, but the information is meager. The major purposes of the current study were to investigate the effects of distractions on problem-solving behavior and performance in non-verbal problems, and the relationships between distractedness, sex, grade level, ability, conceptual tempo, problem difficulty, problem setting, and problem-solving performance.

Six schools were chosen so as to represent, as far as practicable, the population of grade 1-3 students in a city public school system. Stratified random sampling procedures were used to select 10 groups, matched on sex and ability, from each of grades 1-3 in the sub-population. At each grade level, each group of 12 students was randomly assigned to one of two partitive division problems (with, or without a remainder) in one of five problem settings representing different types and amounts of distraction (minimal, situational, color-attribute, spatial-numerical, or maximum distraction).

The 360 subjects attempted one problem individually, and were asked how many of 12 (or 13) given cars would be on each of three trucks if as many as possible were loaded so as to put the same number on every truck. Each subject was questioned closely in an attempt to

get at the thought processes used in the problem-solving task; in particular, to check for attention to distractions. Verbalizations were recorded on audio-tape, and non-verbal behaviors were noted on data cards. Subjects were allotted a score on the problem-solving task, and were classified as being distracted or non-distracted, and as fast (impulsive) or slow (reflective) responders.

A five-way analysis of variance revealed highly significant main effects on problem-solving performance due to grade, ability, problem difficulty, and problem setting. There was no significant effect due to sex, but some interaction effects were noted. Chi-square goodness-of-fit tests, and tests for differences between proportions or means were used to investigate other relationships. Very few of the subjects used a systematic process. The majority were distracted by irrelevant spatial-numerical or color-attribute cues. Distractedness was influenced by the problem setting, and had a very significant effect on problem-solving performance. The ability to cope with distractions increased with grade level, and was greater among girls than boys. More subjects were distracted in the problem with a remainder than in the one with no remainder. Reflective responders were much more successful than impulsive responders, but conceptual tempo was independent of distractedness, sex, ability, or grade level.

One conclusion was that distracted subjects appear to identify and solve a different problem from the one assigned. Thus, irrelevancies act as distractions only when they form the basis of a plausible alternative problem for the child. Finally, some recommendations were proposed for both teaching and further research.

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CHAPTER I

THE PROBLEM AND THE NATURE OF THE INVESTIGATION

Problem solving has always been an important component of school mathematics programs. Yet, despite extensive research, there is still much to learn about how children solve problems and what factors affect problem-solving performance. There is evidence to show that the inclusion of irrelevant data in a word problem increases its difficulty level. There are also studies which reveal that young children can experience difficulty in formulating specific concepts from concrete and pictorial material due to their attention to irrelevant perceptual cues. However, there is no research which has systematically investigated the role of irrelevancies in non-verbal problems. This study is an attempt to provide a foundation for bridging that gap.

Background to the Study

In the field of mathematics education, problem solving has received more attention by researchers than any other area (Riedesel & Burns, 1973; Suydam, 1976). Reviews of research on problem solving in mathematics have provided some useful guides for teaching and learning, but many questions remain unanswered in this area (Kilpatrick, 1969; Riedesel, 1969; Suydam, 1976). Almost all the research undertaken to date has been directed at verbal problems.

At the University of Alberta a research project was initiated

to study the behaviors of young children, aged 3 to 9 years, in solving non-verbal problems (Nelson, 1976a; Nelson & Sawada, 1975). The use of problems which were largely manipulative in nature meant that children could be studied at a much younger age than is possible in the case of word problems. In addition, since the problems required many overt behaviors, a deeper insight could be gained into how children go about solving problems. A set of criteria for 'good' problems was developed for the study (Nelson & Kirkpatrick, 1975). These criteria were based on the research which was currently available; in particular, on the work of Bruner, Dienes, and Piaget. The list of criteria for good problems is as follows:

1. A problem should be of significance mathematically.
2. The situation in which the problem occurs should involve real objects or obvious simulations of real objects.
3. The problem situation should capture the interest of the child.
4. The problem should require the child himself to move, transform, or modify the materials.
5. The problem should offer opportunities for different levels of solution.
6. The problem situation should have many physical embodiments.
7. The child should be convinced that he can solve the problem, and he should know when he has a solution (Nelson & Kirkpatrick, 1976, pp 71-72).

A number of investigators have reported findings from the above research project (Bourgeois, 1976; Bourgeois & Nelson, in press; Little, 1976; Nelson, 1976a; Nelson & Kieren, 1977). One finding showed that some children seem to experience difficulties in solving

problems due to their attention to irrelevant aspects of the problem situation. For example, Bourgeois found that, in division problems, many of the younger children classified objects according to attributes such as color, size or kind when attempting to make equivalent sets of things. Some children also engaged in detailed simulations which were not central to the problem.

Such irrelevant data are generally referred to as "noise" or "distractions". Skemp (1971) maintains that "the greater the noise, the harder it is to form the concept (p. 29)." He also states that the ability to form concepts under conditions of greater noise is an attribute of higher intelligence. Dienes (1963) makes a similar claim, and sees cutting through noise as a feature of the development of a mathematical concept. Biggs (1968), who develops the notion of assimilating information through coding of the relevant input, claims that "adaptive cognitive development is characterized by the production of highly economical codes that cut away the maximum of noise (p. 26)."

The Piagetian studies provide numerous examples where young children's centrations on irrelevant data prevent them from developing specific concepts. For instance, children who had not grasped the concept of distance considered that the distance separating two objects varied according to whether a door placed between them was opened or closed (Piaget, Inhelder, & Szeminska, 1960, pp 71-83). In the construction of a projected straight line, many children in the pre-operational stage saw the line as being straight only when it was parallel to the edge of the desk (Piaget & Inhelder,

1963, pp 156-169). Children were often unable to conserve because they focused on some irrelevant attribute; for example, a set of objects would be considered by some non-conservers to contain more when the objects were spread out further (Piaget, 1952, pp 43-47). Piaget's (1969) experiments with children's concepts of time showed that children were easily distracted by both speed and distance in reference to the notion of 'the same time' (pp 98-109). Inhelder and Piaget (1964) found that very young children continually varied their criteria when they attempted to classify objects (pp 21-31). Vygotsky (1962) also found similar results in tasks involving the classification of blocks -- children changed criteria to whatever attracted them.

Stevenson (1975) lists a number of key factors of relevance to learning and cognition in mathematics. Two of these are repeated below.

> Children may make errors because they attend to the irrelevant attributes of a situation (p. 4).
>
> Young children are easily distracted by the presence of irrelevant information (p. 5).

Stevenson quoted several studies to support these statements. Gelman (1969) used oddity tasks to train five-year-old non-conservers to attend to the numbers of dots and to ignore the irrelevant configurations of the dots on task cards. Following this training nearly all the subjects showed perfect conservation of both number and length. More than half were also able to conserve liquid and mass. In a similar study Bryant (1974) succeeded in training four- and five-year-olds to conserve number. Objects were arranged in a line and the

subjects were taught to ignore the irrelevant length cue. Lubker and Small (1969) presented young children with oddity tasks which required them to select the object different in color. When one or two irrelevant attributes such as size or thickness were also included, the subjects performed only slightly above chance level. With no irrelevant attributes, over 90 percent of responses were correct.

Mansfield (1970) used three different visual discrimination problems to investigate the effect of noise on the performances of kindergarten and first-grade children. On each problem the subject learned to make a correct response in the absence of noise. Irrelevancies were then added in graduated increments in successive trials until the subject made an error. The thresholds for noise were found to be a function of the type of problem. Also, there was an improvement with age in the ability to cope with noise.

There are also other studies which indicate that young children are distracted by visual cues. For example, Saltz et al (1972) administered classification tasks to children in kindergarten, grade three, and grade six. The subjects had to decide which of 70 pictures were instances of six different concepts. The physical appearance of objects was a more critical factor in the conceptual behavior of the younger children than the older children. Thus, younger children focused on the irrelevant perceptual attributes rather than on the functional ones. Winer and Kronberg (1974) presented class-inclusion tasks in both verbal and pictorial form to children in grades K-6. The purely verbal form of question was found to be less difficult than the pictorial form. A similar study

by Wohlwill (1968) yielded the same findings. Smedslund (1964) also obtained similar results in a study involving the class-inclusion concept. Children were introduced to materials set before them, then asked a class-inclusion question about these materials. Smedslund found that his subjects were more likely to respond correctly when the objects were covered over than when they were left exposed. Apparently, certain visual cues tended to act as distractions.

Bruner (1966) identified three modes in which information is represented and processed -- the enactive, iconic, and symbolic modes (p. 28). These correspond to the concrete, pictorial, and abstract levels of representation. The results of the four studies just cited suggest that young children perform better in the symbolic mode than in either the iconic or enactive modes. This may be true for certain concepts. However, it is contrary to what is advocated in mathematics learning generally, and conflicts with other research. For example, Steffe and Johnson (1971) found that first-graders performed better on addition and subtraction problems when objects were present than when they were not. This result applied to all eight different problem types used in the study.

A number of studies have been undertaken to investigate the effect of distractions on performance in word problems. Jerman (1972) used a linear regression model in an endeavor to predict fifth-graders' performances on verbal problems in mathematics. He found that 87 percent of the variance in problem difficulty was accounted for by five of nineteen selected independent variables. One of the five variables was a verbal distraction. The verbal distractions

were in the form of misleading cue words such as "more" or "left" which actually had no bearing on the solution of the problem. Nesher and Teubal (1975) devised word problems which could be solved by either addition or subtraction and presented them to first-graders. When the cue word "more" was included in the problem subjects tended to use addition to solve it, but when the cue word "less" was included in the problem subtraction was the preferred operation. The same results were obtained with fifth- and sixth-graders using the cue words "buying" and "losing". As Nesher and Teubal point out, such cue words are sometimes relevant and at other times not.

Biegen (1971) defined three types of data in verbal problems he devised for eighth-graders -- necessary and sufficient data to solve the problem, irrelevant data which could not be used to solve the problem, and immaterial data which could only lead to a solution through the use of inefficient strategies. Biegen found that problems which included only significant data were the least difficult; problems which also contained irrelevant data were more difficult; and those with immaterial data were the most difficult. Blakenship and Lovitt (1976) cited several studies which demonstrated that problems containing extraneous information severely impaired the accuracy of educable mentally retarded (EMR) students. Blakenship and Lovitt undertook an intensive study of seven boys, aged nine to twelve years, of normal IQ who were functioning one year behind their peers in mathematics. They found that both the presence and the placement of extraneous information in word problems involving addition and subtraction affected both speed and accuracy of computation. Sedlak (1974) undertook a

similar study with nine-year-old EMR children. His study showed that both good and poor problem solvers found word problems with extraneous information more difficult than problems without such information.

It is apparent that children can be distracted in all three modes of representation -- the enactive, iconic, and symbolic. The Piagetian studies provide many examples of children being distracted by perceptual cues at the pre-operational stage. However, it would be reasonable to assume that when children attain the stage of formal operations distractions will have little effect. There is some evidence to support this assumption. Mansfield (1970), in his review of the literature in this area, noted that the amount of incidental learning seems to decrease as children get older. This decrease in incidental learning could be interpreted as a decrease in attention to irrelevancies. Keele (1973) cited several studies with adults which showed that irrelevant attributes either had no significant effect on performance, or caused some interference only when they were very difficult to discriminate from relevant cues.

Arter and Clinton (1974) examined some effects of irrelevant data in arithmetic word problems for fourth-graders. Problems were administered individually with multiple-choice responses provided. Irrelevant data had no significant effect on errors but influenced the time taken to complete the problem. The problems with irrelevant data took longer to solve than problems without such data.

The study by Arter and Clinton raises questions concerning the relationship of conceptual tempo to problem-solving performance, and to how children handle distractions in problems. Kagan (1965) used

his "Matching Familiar Figures" test to identify (fast) impulsive and (slow) reflective subjects. Cathcart and Liedtke (1969) used similar procedures to classify second- and third-graders. They found that reflective subjects performed significantly better than impulsive subjects on both basic number facts and problems. Schwebel and Schwebel (1974) administered class-inclusion and conservation items to a control and an experimental group of first- and second-graders. The subjects in the experimental group were not permitted to respond until a given period of time had elapsed. This group performed significantly better than the control group where subjects could respond at will. It seems that conceptual tempo has considerable influence on children's performances on mathematical tasks. It would also be useful to know whether or not impulsivity and reflectivity are related to how children handle distractions in mathematical problems.

It is apparent from the research already discussed that children can be distracted by irrelevant data. Perceptual cues can act as distractions in both enactive and iconic modes of representation. Piagetian and related studies have shown that attention to irrelevant attributes hinders the development of specific concepts. Investigations of distractions in problem solving have been restricted to the symbolic mode of representation. There is a need to learn more about the role of distractions in non-verbal mathematical problems. This study was prompted by that need. Bourgeois (1976), in his study of young children's behaviors in non-verbal division problems, made this recommendation for further research: "The area most in need of

answers at the present time appears to be the role that distractions play in problem-solving situations (p. 116)."

Although the study reported by Bourgeois was not specifically designed to investigate distractions it did provide some useful hints on possible lines of inquiry. He found that young children often classified animals by kind and cars by color when they were required to make equivalent sets of objects. In one division problem requiring three cars to be put on a ferry-boat some children wanted to load four cars simply because the ferry had room for four. This suggested what may be termed a "spatial-numerical" distraction. The realism of the problem situation seemed to encourage detailed simulations which were not relevant for solving the problem. These findings suggested at least three types of distractions -- color-attribute, spatial-numerical, and situational.

In commenting on these results, Nelson (1976a) stressed the need to develop

> problems in division in which the child could be provided with distractions of various kinds and amounts and to make a systematic study of these distractions and their influence on problem solving behavior (p. 52).

For this study it was decided to devise five different problem settings involving division; one with minimal distraction, one for each of the three distraction types described above, and one to include all three kinds of distractions. In this way, the effects of different qualities and quantities of distractions could be investigated.

Kilpatrick (1969) suggested that researchers of problem solving in mathematics should "undertake clinical studies of individual subjects . . . because our ignorance in this area demands clinical

studies as precursors to larger efforts (p. 532)." The Soviets have found this approach very fruitful in their studies (Kilpatrick & Wirszup, 1969). The use of qualitative research methods and the interview technique to determine how children think are currently receiving increased support (Nelson, 1976b; Suydam, 1976). The interview technique seems to produce valuable information which might otherwise be missed, as shown by Lankford (1974), Erlwanger (1975), and Ginsburg (1976).

In this study the interview was chosen as one method for investigating the role of distractions in the non-verbal problems. The use of such problems meant that some information could also be gained by observing overt actions generated by the problem setting. Piaget (1973) states that "the pupil will be far more capable of doing . . . than of expressing himself verbally (p. 86)." However, Shulman and Elstein (1975) warn that observations of actions are insufficient in themselves. They maintain that it is probably more crucial to note how the subject "sizes up the situation, how the problem is formulated, what is judged to be relevant and what irrelevant (p. 36)."

The thinking aloud technique has been widely used in problem-solving studies. However, Donaldson (1963) points out that such a procedure may interfere with and actually alter thought processes. There is some evidence to support this contention (Flaherty, 1975; Shulman & Elstein, 1975). The thinking aloud technique was therefore rejected as a procedure for this study. Shulman and Elstein claim that the use of stimulated recall to review the problem could yield

valuable insights into the problem-solving process. Success with this procedure would probably be somewhat limited in the case of younger children. However, for the current study it was considered to be a useful complement to the direct observation of problem-solving behaviors.

In this study, scores were allocated to each subject on the problem-solving task in order to differentiate between subjects' performances in a quantitative manner. The general trend in school mathematics and in most mathematics tests has been to score items either right or wrong. However, there is more to mathematics than simply producing the correct solution; especially in the case of problem solving. Polya (1957), in emphasizing the heuristics of problem solving, suggested four main phases: understanding the problem, devising a plan, carrying out the plan, and looking back. Gagné (1966) summarized the main stages of problem solving as follows: "(1) statement of the problem; (2) defining the problem, by distinguishing essential features; (3) searching for and formulating hypotheses; (4) verifying the solution (p. 138)."

It appears that success on a problem-solving task should not be measured on the basis of the solution alone. Indeed, it is possible to attain a correct solution by an invalid process. Also, incorrect solutions can result from miscalculations even when a valid process is used. Then again, a subject may employ a valid process and solve the problem yet still be unsure of the result. Thus, it seemed appropriate that three factors should be considered when allocating scores on the problem-solving task: the process used, the

solution, and verification of the solution.

A pilot study was undertaken to refine the procedures to be followed for the main study (Bana & Nelson, in press). This is discussed in Chapter II.

Purpose of the Study

The central purpose of this study was to investigate the effects of distractions on young children's problem-solving behavior and performance. More specifically, the major purposes were to:

(i) observe, record, and analyze behaviors of first-, second- and third-graders attempting to solve one of two non-verbal partitive division problems (one with a remainder; the other with no remainder) in five problem settings representing different qualities and quantities of distractions;

(ii) investigate subjects' interactions with distractions in the five problem settings;

(iii) examine the effects of distractedness and conceptual tempo on problem-solving achievement, and the relationships of these factors to sex, ability, grade level, problem difficulty, and problem setting; and

(iv) determine the main effects of sex, grade, ability, problem difficulty, and problem setting on problem-solving achievement, and also determine any interaction effects.

Definitions

Conceptual Tempo. The tendency of a subject to respond either quickly or slowly to the problem question. Based on a median split of response latency only, subjects were classified as either "impulsive" (fast responders) or "reflective" (slow responders) on the dimension of conceptual tempo.

Distractedness. Whether or not a subject attends to distractions in the problem setting.

Distraction. Any item of information which is not relevant to the development of a concept or the solution of a problem.

Partitive division. A partitive division problem is one in which the number of elements in each subset is to be determined when a set of elements is separated into a given number of equivalent subsets.

Problem setting. The characteristics and arrangement of the apparatus designed to embody a problem.

Problem-solving behavior. The overt actions and verbalizations manifested by a subject's interaction with the problem setting during his or her attempt to solve the given problem.

Assumptions

1. It was assumed that the two problems embodied in any one of the five problem settings devised for the study each met the set of criteria for a good problem.
2. It was assumed that the behaviors of the subjects in a clinical

situation were indicative of the way they would tackle the problem in their everyday experiences.

3. It was assumed that the range of observed problem-solving behaviors were dictated by the problem and the problem setting.
4. It was assumed that the effects of distractions, such as the use of a tape-recorder, which were extraneous to the problem setting were the same for all subjects.
5. It was assumed that from the recorded data it was possible both to allocate meaningful scores to each subject on the problem-solving task, and to determine whether or not each subject was distracted.

Limitations of the Study

1. No attempt was made to investigate the effects of possible distractions in the problem question which was posed verbally by the experimenter. However, the problem question was identical for all subjects irrespective of the problem or the problem setting.
2. The problem settings did not contain all possible distractions for the two partitive division problems.
3. Since both problems involved partitive division, and each subject attempted only one of these, the effects of distractions in these problem settings would have limited generalizability to other mathematical problems.
4. Although it is possible that the socio-economic status (SES) of subjects may have some effect on how they deal with distractions, SES was not used as an independent variable in this study. Instead,

an attempt was made to select a sample which would reflect the distribution of SES levels in the population.

5. The procedure devised for the allocation of performance scores on the problem-solving task was not based on empirical research, and therefore may not have been the most appropriate system to use.
6. A section of each interview was largely unstructured. This led to some loss of information in categories where the data were to be analyzed in a quantitative manner. Also, in some instances, questions were posed in such a manner that responses were suggested to the subjects, so the information gained in this way could not be interpreted.

Significance of the Study

It has been shown that distractions significantly affect the development of specific concepts when enactive and iconic modes of representation are used. The inclusion of irrelevant data in mathematical word problems tends to increase the difficulty of such problems. However, the role played by distractions in non-verbal mathematical problems is not at all clear, and raises many questions. This study which is limited in scope cannot provide all the answers, but it should yield useful information which may form the basis for more extensive research in this area.

During the past two decades there has been an increasing emphasis in school programs on the development of mathematical concepts from realistic situations within the child's environment. This

trend has been accompanied by the use of a wide variety of both structured and unstructured manipulative aids, particularly in the lower elementary school grades. Freudenthal (1973) insists that "mathematics should be tied to reality when it is learned (p. 405)." Most mathematics educators would agree. But in order to abstract mathematics from the environment the child needs to cut through many irrelevancies. More needs to be known about the effects of these distractions on mathematics learning so that the teacher will be in a better position to make effective use of realistic problems with concrete materials.

Outline of the Study

The problem and the nature of the investigation were explained in this chapter. Chapter II contains a detailed description of the design of the investigation. The findings of the study are reported in Chapter III. Chapter IV includes a summary of the investigation, a discussion of the findings and their implications, a list of conclusions, and recommendations for both research and teaching.

CHAPTER II

DESIGN OF THE INVESTIGATION

The central purpose of the study was to determine the effects of distractions on young children's problem-solving behavior and performance in non-verbal problems; namely, partitive division problems. The specific purposes were listed in Chapter I. This chapter provides details of the problems and problem settings, the population and sample, the interview protocol and scoring procedure, the pilot study, and the methods employed to analyze and verify the data.

The Problems

Two partitive division problems were devised for the study, and each conformed to the set of criteria for good problems outlined in the previous chapter. One problem involved three toy trucks and 12 toy cars. The subject was shown the materials and asked how many of the given cars would be on each truck if as many as possible were loaded onto the three trucks so there would be the same number of cars on each truck. This problem was defined as one of low difficulty, and was designated problem A. The second problem, designated problem B, was identical to problem A except that there were 13 cars instead of 12. Problem B was defined as one of high difficulty, since it involved a remainder.

The two problems may be expressed mathematically as follows:

Problem A. $12 \div n = 3$

Problem B. $13 \div n = 3$

The algorithmic form for division may also be used to express the problems.

Problem A. $12 = (3 \times n) + r$

Problem B. $13 = (3 \times n) + r$, where n and r are whole numbers and $0 \leq r < 3$.

Since one problem involves a remainder, and the study involves young children, the second method of representation seems more appropriate.

The Problem Settings

Each of the two problems was embodied in five different problem settings. These settings were designed to include different types and amounts of distractions. The first problem setting, which is illustrated in Plate 1, was defined as the "minimum distraction" setting. The manipulative materials consisted of three yellow toy trucks and 12 yellow toy cars (13 for problem B). All three trucks were identical in shape, color, and size. Each truck tray was 11.2 cm long and 6.5 cm wide. The 12 (or 13) cars were similar in color, size, and basic design, with very slight variations in styling. All the cars were 5.3 cm long and about 2 cm wide. Thus, each truck could hold six cars if they were placed lengthwise and packed close together, or four cars with reasonable spacing; or five cars if they were placed crosswise on the truck. These manipulative materials were

Plate 1. First Problem Setting

arranged as in Plate 1 on a pale green sheet of paper 97 cm x 64 cm which was on a low table.

The materials used in the remaining four problem settings had the same dimensions as those in the first setting. The second problem setting, which is illustrated in Plate 2, was defined as the "situational distraction" setting. The paper backing sheet had a roadway and parking area painted on it, and all other objects were arranged as in Plate 2. The cars and trucks were the same as in the first problem setting. A model of a building was located across the road from the parking area which contained the 12 (or 13) cars. The three trucks were on the roadway, with the first truck at a loading ramp. A toy man was positioned in the right front corner on the back of each of the first two trucks in line, while another man was

Plate 2. Second Problem Setting

Plate 3. Third Problem Setting

near the foot of the loading ramp. The additional objects in this setting were included to add more realism to the problems, and to act as possible distractions.

The third problem setting was the "color-attribute distraction" setting as shown in Plate 3. The first truck in line was blue, the second was yellow, and the third one red. There were three blue cars, four yellow cars, and five (or six) red cars. The cars were arranged as shown so that no two cars of the same color were adjacent to one another. The fourth problem setting included the same materials as the minimum distraction setting except that six bays were marked on the back of each truck, as illustrated in Plate 4. The fourth setting was defined as the "spatial-numerical distraction" setting.

Plate 4. Fourth Problem Setting

The fifth problem setting pictured in Plate 5 was called the "maximum distraction" setting, since it included all the irrelevancies of the other problem settings. All the materials were positioned in identical fashion to the arrangement in the second situational distraction setting. Both the trucks and the cars were color sequenced as in the color-attribute distraction setting, but the backs of the trucks were marked out in six bays as in the fourth problem setting. The five photographic plates illustrate the five different settings for problem A. Problem B was embodied in the same five settings; the only difference being the inclusion of one additional car in each case.

Plate 5. Fifth Problem Setting

Population and Sampling

The population for the study consisted of the children in grades one, two, and three in the Edmonton Public School system in Edmonton, Alberta. It was not possible to gain access to all schools, so the sample could not be drawn from the population as a whole. Therefore a modified procedure was used. Six elementary schools were selected from those in the system, and the sample of subjects was drawn from them. The selection of the schools was based primarily on socio-economic status (SES) but also on geographic location, in order to gain some measure of control over these two variables. The SES of schools, rated on a five-point scale ranging from one (for very low) to five (very high), was obtained from the school authorities. Four schools with a rating of three, one with a rating of two, and one with a rating of four were selected. This distribution of ratings reflected -- as near as possible -- the distribution for the system as a whole. The six schools were also selected so as to represent the major geographic areas of the city. These schools, which were comparatively large, were considered to provide a sample which was reasonably representative of the total population of grade one, two, and three children.

Most of the children in the six selected schools were organized into single-grade classes. The first-graders in these schools were in 25 different classes, the second-graders in 22, and the third-graders in 20 different classes. A stratified random sample of 120 children based on sex and ability was drawn from each

of the three grade levels, thus providing a total of 360 subjects for the study. No single criterion measure for ability was available for all three grades, so two different measures were used. A subject's percentile rank on the Metropolitan Readiness Test (MRT) was used as the criterion for ability in grade 1. In each of the other two grades a subject's percentile rank on a standardized mathematics test, developed and administered by the school system, was the ability criterion.

The procedure used to select subjects was as follows. Ten boys were randomly selected from all first-grade boys in the six schools who had MRT percentile ranks in the 10-15 range. These subjects were randomly assigned to ten groups on the basis of one per group. This process was also used to select and allocate ten first-grade girls from the 10-15 percentile range, resulting in a boy-girl pair at this ability level in all ten groups. The same procedure was employed for each of the following percentile ranges: 25-30, 40-45, 55-60, 70-75, and 85-90. Thus, all ten first-grade groups were matched on sex and ability. Each group consisted of three boys and three girls above the median score on the MRT (high ability), and three boys and three girls below the median score (low ability).

Ten matched groups for each of grades two and three were selected in the same way, except that the percentile ranges applied to standardized mathematics tests and not to the MRT. At each grade level each group of 12 subjects was randomly assigned to either problem A or problem B in one of the five problem settings. Thus,

each of the 360 subjects attempted only one problem.

Interview Protocol and Scoring Procedure

The researcher, in the role of experimenter (E), interviewed every subject (S) individually and used the following protocol for all subjects:

> The materials for the particular problem and problem setting were arranged as shown in the photographic plates and placed on a low table in an interview room. E walked with S from the classroom to the interview room. During this walk E conversed informally with S to make him or her feel at ease, then stated: "I just want you to try something for me." S was shown the cars and trucks then asked whether he or she had toy trucks at home, and whether S ever played with them. E, who sat across the table from S, then posed the problem question as follows: "We have to load as many of those cars as we can onto the three trucks, but we must put the same number of cars on every truck. How many of those cars will be on each truck? I'll ask you again to make sure you understand." E then repeated the problem question and asked: "Do you understand?" E repeated the problem question again if S was still unsure, if S asked any questions at all, or if S seemed uncertain how to respond. When S gave a solution E asked: "Are you sure that's right?" If S responded in the affirmative E asked again: "Quite sure?" Otherwise S was free to check the solution or alter it. In any case the previous two questions were repeated for each solution given, until S either confirmed a solution or indicated the unlikelihood of any further progress. When a solution was confirmed E asked a question of the following form:

> "How did you know there would be . . . on each truck?" S was then questioned further about his or her actions, verbalizations, and thoughts to determine the process used to solve the problem, whether or not S was confident of the solution if it was correct, the reasons for any observed actions, and S's reactions to distractions.

The above interview protocol was followed for all 360 subjects. Care was taken to ensure a standardized format during the period from when the problem question was first posed until the subject confirmed a solution and was asked the reason for it. Thereafter the interview session was unstructured, and was determined by the subject's antecedent and subsequent verbal and non-verbal behaviors. It was therefore inappropriate to follow a set interview pattern once the problem-solving task was complete. The major objective was to get at the thought processes used by the subject in attempting to solve the problem. Each interview was recorded by means of a small cassette tape-recorder with an internal microphone. The subject was observed closely, both while attempting to solve the problem and during the subsequent interview session. Non-verbal behaviors were noted and recorded on a data card.

Each subject was allocated a score of either zero, one, two, or three points for the problem-solving task. One point was awarded if the subject's first solution was correct. Whenever a solution was given the subject was asked whether he or she was sure this was correct. Thus, opportunities were provided for subjects to reflect on their responses and alter them if they desired. A point was awarded if the subject confirmed the correct solution and

demonstrated confidence in it. The third point was allotted if the subject used a valid process in attempting to solve the problem. If the subject made a deliberate attempt to partition the given set of cars into three equivalent sets this was considered to be a valid process. The following three methods were each classified as a valid process for both partitive division problems:

(i) Using division or multiplication operations;

(ii) Distributing one car per truck in rotation; and

(iii) Employing trial and error methods, either manipulative or otherwise, in attempting to partition the given set of cars into three equivalent sets.

The use of any one of the above processes could lead to the solution of the problem.

It was possible for a subject to give the correct solution initially by some invalid process, such as deciding that only four would fit on each truck, then failing to confirm this solution. Only one point was allocated in such cases. Those who also used a valid process were obviously more successful on the problem-solving task and were awarded an additional point for this. However, to gain the third point subjects had to show mastery of the task by confirming their solution and demonstrating confidence in it. A subject who used a valid process but did not attain the correct solution was awarded one point only. If a subject gave an incorrect solution initially, but then achieved solution by a valid process and confirmed it, two points were allotted.

The average time taken for each interview was approximately five minutes. All the data for the study was collected in

November and December, 1976.

The Pilot Study

In October, 1976 a pilot study was undertaken to check the feasibility of the main study and clarify procedures to be used (Bana & Nelson, in press). Using a similar procedure to that outlined for the main study, four groups matched on ability were selected from each of grades one, two, and three in a large elementary school in a relatively high socio-economic area of Edmonton. Each group consisted of four boys and four girls. At each grade level each of the four groups was randomly assigned to problem A ($12 \div n = 3$) in one of the first, second, third, or fourth problem settings which were described previously. The interview protocol and scoring procedures were similar to those adopted for the main study. However, the problem question was put in a slightly different manner and began with, "We have to load all those cars onto the three trucks". This was altered to "We have to load as many of those cars as we can onto the three trucks" for the main study, due to the inclusion of problem B ($13 \div n = 3$) which involved a remainder.

Almost one half of the 96 subjects in the pilot study were distracted by irrelevancies in the problem settings. Subjects who attended to distractions were less likely to attain solution of the problem than those who ignored them. The mean scores for all three grades combined were 2.08, 1.83, 1.33, and 1.17 in the four respective problem settings. The fourth setting, with bays marked on the trucks to act as a spatial-numerical distraction, produced

the lowest mean score. There were some significant differences between scores in the four problem settings, both within each grade and over all three grades. Thus, the problem settings which represented different qualities or quantities of distractions affected scores on the problem-solving task. In the case of subjects who solved the problem verbally without manipulating any of the cars, it was sometimes difficult to determine whether they had been distracted or not. Also, distracted subjects rarely gave any reason for their strategy.

The pilot study showed that distractions do play an important role in young children's problem-solving behaviors, and provided justification for a more extensive study. The design of the pilot study provided a viable rationale for the main study, with some modifications. Firstly, a fifth problem setting to include a number of distractions in combination seemed warranted. Secondly, the pilot study's results suggested that if children are distracted differently in different problem settings then perhaps the same might be true for different levels of problem difficulty -- thus, the inclusion of problem B in the main study. Finally, the interviewing experience in the pilot study pointed to the need to question children more closely in order to get at the thought processes used in their attempts to solve the problem.

Data Analysis Procedures

Individual data cards were prepared prior to the interview. Each card identified the subject and indicated the sex, grade,

ability level, problem difficulty level, and problem setting for that subject. Non-verbal behaviors exhibited during the problem-solving task and interview were recorded on these data cards. Verbalizations were recorded on audio-tapes. When the data collection was complete categories were set up to summarize and classify the data from both the cards and tapes. The format of the data coding sheet, including some examples of coded data, is given in Appendix B.

The various methods of solution used by subjects, including subjects' attention to distractions, and the rationales behind the different solution strategies were analyzed and described. The time which elapsed from when the problem question was repeated until the subject gave his or her first solution was noted. This was used to classify subjects on conceptual tempo as either fast or slow responders, based on a median split. The relationships between factors such as methods of solution, distractedness, and conceptual tempo; the effect of these factors on scores; and the relationships of these factors with sex, grade, ability, problem difficulty, and problem setting were examined by chi-square tests of goodness of fit, or z tests of differences between proportions or differences between means as appropriate. A five-way analysis of variance was carried out to test for the main effects of sex, ability, grade, problem difficulty, and problem setting, and for any interaction effects. The studentized range statistic or q test was the a posteriori test used on the results of the ANOVA to check for significant differences between means within single categories (Winer, 1971, pp 185-187).

Experimenter Reliability

The experimenter coded the data from the audio-tapes and data cards onto a coding sheet, as illustrated in Appendix B. An independent coder was engaged to do likewise for a sample of subjects in order to establish some estimate of reliability. The independent coder was instructed in the coding procedure to be used, and the categories on the coding sheet were explained to her. She then coded the data from cards and tapes for five subjects not included in the reliability check. These results were then compared with those of the experimenter, and points of agreement and disagreement were identified in order to establish consistent use of the coding system.

A stratified random sample of 30 subjects was used for the reliability check. One subject was drawn at random from each of the ten matched groups in each of the three grade levels. When the independent coder had completed coding the data for all 30 subjects her results were compared with those of the experimenter. Arrington's method of computing the coefficient of agreement between observers was used (Bott, 1933, p. 67). The coefficient of agreement was 0.94. A specific comparison of the scores awarded was also made. Only two of the thirty scores differed -- one by one point, and the other by two points -- thus resulting in a reliability coefficient of 0.93. Based on these results, experimenter reliability was considered to be adequate for the purposes of this study.

CHAPTER III

RESULTS OF THE INVESTIGATION

Ten matched groups were selected from each of grades one, two, and three in six metropolitan schools. Each group consisted of three boys and three girls from the high ability range, and three boys and three girls from the low ability range. At each grade level each of the ten groups was randomly assigned to one of two non-verbal partitive division problems in one of five different problem settings. The major purposes of the study were to:

(i) observe, record, and analyze behaviors of first-, second- and third-graders attempting to solve one of two non-verbal partitive division problems (one with a remainder; the other with no remainder) in five problem settings representing different qualities and quantities of distractions;

(ii) investigate subjects' interactions with distractions in the five problem settings;

(iii) examine the effects of distractedness and conceptual tempo on problem-solving achievement, and the relationships of these factors to sex, ability, grade level, problem difficulty, and problem setting; and

(iv) determine the main effects of sex, grade, ability, problem difficulty, and problem setting on problem-solving achievement, and also determine any interaction effects.

The results of the investigation are reported in this chapter under the following headings:

Methods of Solution

Rationales for Solution Strategies

Distracted Subjects

Distractedness and Criterion Scores

Distractedness and Conceptual Tempo

Relationships of Distractedness to Other Factors

Results of Five-Way ANOVA

The Effect of Problem Setting and its Relationship to Other Factors

Interaction Effects on Scores

Major Findings

The first section, which follows, deals with methods of solution.

Methods of Solution

In the interview protocol adopted for the study there was no reference to the method to be used to solve the problem. Thus, each subject had to decide whether or not to manipulate the materials without any specific directions from the experimenter. Subjects who gave verbal responses without manipulating any of the toy cars were classified as verbal solvers, while those who did manipulate the cars were classified as manipulative solvers. Manipulative behaviors always consisted of loading cars onto trucks, except for two cases where the cars were arranged in groups on the table but were not put on the trucks. Many of the subjects who loaded the cars to arrive at a solution confirmed their result verbally when asked to do so, or

altered their solution, while some carried out further manipulations before settling for a particular response. Others gave a verbal response initially but, when asked to confirm it, they loaded the cars to determine the solution. In all these cases the individuals were classified as manipulative solvers. The frequencies of manipulative solvers in the five problem settings are shown in Table 1.

Table 1

Frequency Distribution of Manipulative Solvers Among Boys and Girls in the Five Problem Settings

Sex	Problem Setting					
	1	2	3	4	5	1-5
Boys	17	18	14	10	5	64
Girls	10	9	17	9	7	52
Totals	27	27	31	19	12	116

Almost one third of the 360 subjects were manipulative solvers, including 36 percent of the boys and 29 percent of the girls, with the difference between proportions of boys and girls being slight ($z = 1.35$, $p < 0.09$). Thirty-four boys and 26 girls from the high ability group and 30 boys and 26 girls from the low ability group were manipulative solvers, so the method of solution was not affected by ability. Neither was it affected by problem difficulty. The number of subjects who manipulated cars was 58 in both problem A (no remainder) and problem B (with a remainder). Of the 116 manipulative solvers, 45 were in grade 1, 37 were in grade 2, and 34 were

in grade 3. Although there appeared to be a tendency for manipulations to decrease in the higher grades the difference between observed and expected frequencies was not significant. However, almost all the difference due to sex was accounted for in the first grade where 29 of the 45 manipulative solvers were boys. A goodness-of-fit test of observed and expected frequencies of verbal and manipulative solvers in the five problem settings showed a significant disparity ($\chi^2 = 14.63$, $p < 0.01$). More than the expected number of subjects in the first three problem settings were manipulative solvers, while less than the expected number manipulated cars in the fourth and fifth problem settings.

Some manipulative solvers engaged in simulations which did not contribute in any way to the solution of the problem. These unnecessary simulations consisted of one or more of the following behaviors: rotating cars so that they all faced the same way on the truck; driving cars up and down the loading ramp; driving trucks around; moving the loading ramp from truck to truck; moving the toy men around unnecessarily; and matching the configurations of the cars on all three trucks. The frequencies of manipulative solvers who engaged in such simulations are shown in Table 2. More manipulative solvers carried out unnecessary simulations in the second and fifth than in the other three problem settings. The proportions of 38 percent and 9 percent respectively were significantly different ($z = 3.81$, $p < 0.0002$). The second and fifth problem settings were more realistic in that they included a building, a loading ramp, three men, a roadway, and a parking area. Unnecessary simulations were more prevalent among

Table 2

Frequencies of Manipulative Solvers Engaged in Unnecessary Simulations

Grade	Problem Settings		
	1,3,4	2,5	1-5
1	4	9	13
2	2	4	6
3	1	2	3
1-3	7	15	22

younger children than among older children. Thirteen subjects exhibited this behavior in grade 1, six in grade 2, and three in grade 3. Only five of the 22 manipulative solvers who engaged in these simulations were girls.

A subject's first response to the problem question indicated a specific arrangement of cars on the three trucks. At this point he or she was asked: "Are you sure that's right?" Some subjects then confirmed their initial response, while others generally tried one or two more arrangements of cars before confirming a particular response. Subjects whose initial or confirmed solution was correct were asked if they had tried other arrangements. This was not always done for subjects with no correct solution. All manipulative solvers revealed arrangements of cars by their actions, although some may have thought about other possibilities as well. Most verbal solvers who gave no correct solution tended to reply impulsively. In such cases they would have been unlikely to attempt arrangements other

than the ones they indicated. All subjects were questioned regarding the process they used in attempting to solve the problem. During this interview session some subjects revealed other arrangements of cars they had tried. A sample of interview transcripts is included in Appendix A.

The number of different arrangements attempted by individual subjects ranged from one to five, as shown in Table 3. Sixty-three percent of the subjects tried only one arrangement, 29 percent tried two different arrangements, and the remainder tried more. No more than three arrangements were attempted in the fourth and fifth problem settings. In fact, the tendency here was to try only one

Table 3

Frequency Distribution of the Number of Different Arrangements of Cars Attempted in Each Problem Setting

Number of Different Arrangements Attempted	Problem Setting					
	1	2	3	4	5	1-5
1	45	35	33	63	51	227
2	18	27	35	8	15	103
3	6	5	2	1	6	20
4	3	4	2	–	–	9
5	–	1	–	–	–	1
Total	72	72	72	72	72	360

arrangement. This was particularly true in the fourth problem setting where 63 of the 72 subjects tried only one arrangement of cars when attempting the problem. In both these settings the trucks had bays marked out on the back of them and most subjects simply put a car in each bay or counted the number of bays. The rationales for the various solution strategies are discussed in more detail later.

Frequencies of specific arrangements or sequences of arrangements of cars are shown in Table 4. Any sequence with a total frequency of five or more is listed in the table. Other types of sequences are categorized together as "Others", which in most cases were attempted by only one or two individuals. A sequence of arrangements such as "3,4,5; 4,4,4" indicates three cars on the first truck, four cars on the second, and five cars on the third truck in line for the first arrangement, followed by four cars on each truck for the second arrangement. The relevant information was obtained from the subjects' actions and their responses to the problem question, and also from responses to interview questions about the processes used when attempting the problem.

The most prevalent sequence involved the one arrangement 6,6,6 which was used by 86 subjects. However, 78 of these were in the fourth and fifth problem settings where many subjects counted the six bays marked on each truck. The sequence involving the single arrangement of four cars on each truck was used by 52 subjects, including only seven cases in the fourth and fifth problem settings. The sequence 3,3,3; 4,4,4 shows a similar imbalance across the five

Table 4

Frequency Distribution of Sequences of Arrangements of Cars in the Five Problem Settings

Arrangement Sequences	Frequencies in Each Problem Setting					
	1	2	3	4	5	1-5
3,3,3	8	2	6	1	5	22
3,3,3; 4,4,4	4	7	6	1	-	18
3,4,5	-	-	4	-	1	5
3,4,5; 4,4,4	-	-	5	-	-	5
4,4,4	17	15	13	5	2	52
5,5,2; 4,4,4	1	2	1	-	1	5
5,5,3	1	3	1	-	1	6
5,5,5	5	2	2	2	5	16
5,5,5; 4,4,4	-	2	3	-	-	5
5,5,6; 6,6,6	1	1	-	-	4	6
6,6,0	3	-	-	6	-	9
6,6,0; 4,4,4	1	-	2	3	-	6
6,6,1	1	1	1	3	1	7
6,6,6	3	3	2	45	33	86
6,6,6; 4,4,4	2	-	2	1	3	8
9,9,9	3	3	-	-	-	6
Others	22	31	24	5	16	98
Total	72	72	72	72	72	360

settings. Many manipulative solvers put as many cars as they could fit on the first truck and continued in this fashion until they ran out of cars. This generally accounted for the arrangements 5,5,2 and 6,6,0 in the case of problem A, and 5,5,3 and 6,6,1 for problem B. The arrangement 5,5,6 occurred in several sequences and was mostly confined to the second and fifth problem settings where a man was positioned on each of the first two trucks in line. Sequences including 3,4,5 for problem A and 3,4,6 for problem B occurred only in the third and fifth settings which involved three colors of both cars and trucks. These two unequal distributions of cars were based on the numbers of blue, yellow and red cars respectively. The total number of different sequences of arrangements of cars attempted by all subjects was 86. These sequences included 39 different arrangements of cars ranging from two on each truck to as many as 20 on each truck.

Rationales for Solution Strategies

The rationale behind each arrangement of cars indicated the process being used by subjects to solve the problem. This strategy or process could sometimes be observed directly in the non-verbal actions of subjects, particularly with manipulative solvers. However, verbal solvers also engaged in non-verbal behaviors such as pointing to cars or to the backs of trucks as they counted. Immediately following the confirmation of a solution the subject was asked a question of the following form: "How did you know there would be . . . on each truck?" This focused directly on the process being used, and

was followed by other pertinent questions appropriate to the situation. It was possible for each subject to score a maximum of three points for the problem. One of these was allocated for using a valid process, which included the following three categories: appropriate use of division or multiplication operations; distribution of one car per truck in rotation; or attempting to make three equivalent sets with the given cars by trial and error.

The identified rationales and their distribution over the various arrangements of cars are shown in Table 5. The 11 most common arrangements are specified in the table, while the remaining 28 are grouped together. Only four subjects used division or multiplication to solve the problem, and 11 subjects used the most efficient manipulative method of distributing one car per truck in rotation to ensure an equal number on each truck. There were 185 other deliberate attempts to make three equivalent sets with the given cars. These involved 116 subjects; some of whom tried more than one arrangement. In all, 36 percent of subjects used a valid process.

As far as could be ascertained, the 116 subjects referred to above used trial and error methods in their attempts to partition the cars into three equivalent sets. Some tried several arrangements before arriving at four for each truck. In problem B a number of subjects did not solve the problem even though they did use a valid process. They tried to make three equivalent sets with the 13 cars, apparently unaware that one car had to be excluded. As seen in Table 4, a total of 52 subjects came up with the single arrangement of four cars per truck. A majority of these subjects apparently

Table 5

Frequency Distribution of Rationales for Specific Arrangements of Cars

Rationale	Arrangement												
	3 3 3	3 4 5	3 4 6	4 4 4	5 5 2	5 5 3	5 5 5	6 6 0	6 6 1	6 6 6	9 9 9	Other	Total
Division or Multiplication	-	-	-	4	-	-	-	-	-	-	-	-	4
Distribution of Cars One by One	-	-	-	11	-	-	-	-	-	-	-	-	11
Trial of Equivalent Sets	36	-	-	109	7	3	4	1	1	-	-	24	185
Guess	3	-	-	3	-	-	2	-	-	1	1	2	12
Same Number of Cars as Trucks	6	-	-	-	-	-	-	-	-	-	-	-	6
Spatial-Numerical Cues:													
(a) Amount of Room	5	-	-	9	4	9	7	8	7	9	7	36	101
(b) The Marked Bays	-	-	-	-	2	2	10	10	6	87	-	12	129
Color-Attribute Cues:													
(a) Cars of Each Color	-	12	9	2	-	-	-	-	-	-	-	3	26
(b) Cars of a Particular Color	7	-	-	3	-	-	-	-	-	3	-	-	13
Induced Spatial-Numerical Cue	-	-	-	6	1	-	6	2	1	7	2	2	27
Induced Color-Attribute Cue	2	-	-	2	-	-	-	-	-	-	-	-	4
No Explanation	6	-	-	2	-	-	1	-	-	3	-	1	13
Other	-	-	-	-	-	-	-	-	-	-	-	3	3
Total	65	12	9	151	14	14	30	21	15	110	10	83	534*

*Some subjects attempted more than one arrangement.

used a trial and error method and were therefore successful with their first trial. They denied having tried any other arrangement or process, and either pointed out why there were four for each truck or made statements like the following: "I could see there was going to be four"; or "I just tried four and it worked". Some of the 52 subjects used one of the other two valid processes discussed previously, while the remainder used some invalid process.

Twelve arrangements were based on guesses, at least according to the subjects. However, it is possible that some of these subjects may in fact have used another strategy. Several others who said they guessed revealed after further questioning that some alternative process was behind their response. Six subjects stated that there would be three cars on each truck because there were three trucks. There were 13 arrangements for which no explanation was supplied by the subjects, and six of these involved three cars on each truck.

Many subjects used what were defined as spatial-numerical cues to determine their arrangements of cars on the trucks. These cues were based on the amount of room on each truck or on the bays marked on the back of each truck. Attention to such cues was determined from both verbal and non-verbal behaviors. For example, manipulative solvers who put as many cars as they could fit on the first truck and then continued in this fashion until they ran out of cars generally obtained one of the following arrangements: 5,5,2; 5,5,3; 6,6,0; or 6,6,1. If such an arrangement was not altered the subject was considered to have used a spatial-numerical cue, otherwise the

use of such a cue was only confirmed or rejected during the interview which followed. Verbal solvers indicated the use of a spatial-numerical cue by focusing only on the trucks and not on the cars, and by pointing to count spaces on the backs of trucks. When asked how they knew there would be the given number of cars on each truck, subjects who used a spatial-numerical cue generally gave a reply like one of the following: "I counted the spaces"; "Because there's six squares";"Because there's enough room for five"; or "That's how many can fit".

As shown in Table 5, 129 arrangements were based on the bays marked on the trucks, and therefore occurred only in the fourth and fifth problem settings. Since there were six bays marked on each truck the response of six cars per truck was the most common one among subjects who used the marked bays as a cue. Most manipulative solvers who attended to this cue put six cars on each of the first two trucks in problem A, and did the same in problem B then put the remaining car on the third truck. Others generally put five cars on each of the first two trucks and two on the third truck for problem A or three on the third truck for problem B. However, this only occurred in the fifth problem setting where there was a man positioned in a bay on each of the first two trucks. Arrangements such as 5,5,5 or 5,5,6 were based on the same reason, except for a few cases where subjects miscounted the bays.

Only one subject in each of the fourth and fifth problem settings ignored the bays but still based the arrangement on the amount of space on the backs of the trucks. Except for these two

cases, all the other arrangements in Table 5 which were determined by the amount of room took place in the first three problem settings. Subjects using this rationale came up with a wide variety of arrangements ranging from three to twenty on each truck. The arrangements 5,5,2 and 6,6,0 were only produced by manipulative solvers in problem A, while in problem B manipulative solvers sometimes distributed the cars into 5,5,3 or 6,6,1 arrangements.

In the third and fifth problem settings the first truck in line was blue, the second was yellow, and the third was red in color. There were three blue, four yellow, and five red cars for problem A, while for problem B there was an additional red car. In these two problem settings some subjects based their arrangements of cars on what were termed color-attribute cues. One strategy was to classify all the cars according to color and match colors of cars with colors of trucks. Thus, in problem A 12 subjects stated that there would be, or actually put three blue cars on the blue truck, four yellow cars on the yellow truck, and five red cars on the red truck. Nine subjects used the same rationale to obtain a 3,4,6 arrangement in problem B. The other five instances in this category were due to subjects miscounting the cars of each color. The second strategy was to base the number of cars for each truck on one of the three colors. Seven subjects determined that there would be three cars on each truck because there were three blue cars, three subjects based the number per truck on the four yellow cars, and three subjects said there would be six on each truck because there were six red cars. Three arrangements were based on both color and spaces.

However, the rationale was classified as spatial-numerical in each case since all three subjects considered the amount of room to be the more important reason for their response.

In 27 cases an arrangement was determined by the subjects with no explanation given for it, but then a spatial-numerical rationale was induced in the subject by the form of the experimenter's question. In an attempt to gain more information the experimenter probably influenced the subject. For example, a question like the following might be asked: "Did you try to fill each truck?" If the subject replied affirmatively then the rationale for the given arrangement was classified as an induced spatial-numerical cue. As can be seen in Table 5, induced spatial-numerical cues applied mostly to cases involving four, five, or six cars per truck. The same criteria were used to determine an induced color-attribute cue, and there were four instances of this. Faulty interview techniques also resulted in cases where the arrangement of cars was induced by the experimenter. For example, if a subject said there would be five cars on each truck because only five would fit, this was classified as a spatial-numerical cue for the arrangement 5,5,5. However, the experimenter may then have asked a question like the following: "Did you think about putting the blue cars on the blue truck, the yellow cars on the yellow truck, and the red cars on the red truck?" If the subject replied in the affirmative this implied an induced 3,4,5 arrangement. Such an arrangement was not recorded. However, an induced color-attribute distraction would be noted in this case. This and other types of distractions are discussed in the following section.

Distracted Subjects

As indicated in the previous section, many subjects attended to spatial-numerical or color-attribute cues which were irrelevant to the problem. Such irrelevancies were classified as distractions for this study. Both verbal and non-verbal behaviors were used to determine whether or not subjects were actually distracted. Some distractions were induced by the experimenter during the interview, as described previously. The frequency distribution of subjects, both those actually distracted and those whose distraction was induced, is shown in Table 6. A total of 238 subjects, or 66 percent, were distracted. Distractions were induced in a further 12 percent of the subjects.

The fourth and fifth problem settings were designed to include a spatial-numerical distraction in that there were six bays marked on the back of each truck. Very few subjects ignored this distraction. In the fourth problem setting 64 of the 72 subjects attended to spatial-numerical cues in their efforts to solve the problem, while 62 did so in the fifth setting. The first three problem settings were not meant to include a spatial-numerical distraction, so the backs of the trucks were unmarked. However, many subjects focused on the amount of space on the trucks and were in fact distracted by this spatial-numerical cue. The numbers of subjects distracted in this way were 28, 36, and 18 in the first, second, and third problem settings respectively. The spatial-numerical cue was a much stronger distraction when bays were marked

Table 6

Frequency Distribution of Distracted Subjects

Problem Setting and Problem Difficulty	Distractions						
	Spatial-Numerical		Color-Attribute		Other	Total Subjects Distracted*	
	Actual	Induced	Actual	Induced	Actual	Actual	Induced
1-A	15	3	-	-	-	15	3
1-B	13	4	-	-	-	13	4
2-A	13	4	-	-	-	13	4
2-B	23	4	-	-	-	23	4
3-A	6	3	17	1	-	21	4
3-B	12	5	15	3	-	25	6
4-A	31	-	-	-	-	31	-
4-B	33	-	-	-	-	33	-
5-A	28	3	4	3	1	30	4
5-B	34	1	6	13	-	34	14
Total	208	27	42	20	1	238	43

*These totals do not always correspond to row frequencies because some subjects attended to more than one type of distraction.

on the trucks than when the backs of the trucks were blank. When there were marked bays the majority of subjects either counted the bays or loaded one car per bay with little or no hesitation. In all, 208 subjects were distracted by the spatial-numerical cues. This amounted to 58 percent of the total sample.

A color-attribute distraction was built into the third and fifth problem settings. Ten subjects were distracted by color in the fifth setting, where the spatial-numerical distraction was strong and tended to dominate. However, in the third setting, where the spatial-numerical cue was weaker, there were 32 subjects who were distracted by color. In these two problem settings 13 subjects attended to both color-attribute and spatial-numerical cues, but in most of these cases color was abandoned as a criterion for arranging the cars on the trucks. In the second and fifth problem settings a toy man was stationed in the right front corner of each of the first two trucks in line. Fifteen subjects were distracted by the men on the trucks in the second problem setting, and 22 were distracted by them in the fifth setting. These subjects considered that the men took up space and that they would therefore influence the number of cars to be put on each truck. Also, all 37 of these subjects were distracted by spatial-numerical cues as described above. Therefore, the men on the trucks simply acted as components of the overall spatial-numerical cues.

All the cars used in the five problem settings were similar in size and structure. However, they were styled according to a number of common makes of cars with brand names embossed in very fine

print. One first-grade subject tried to classify the cars according to the brand names. There was no other evidence of subjects being distracted in this way, but one boy pointed out his family's car type during the interview session. In the second and fifth problem settings the men, loading ramp, factory building, roadway, and parking area were intended as situational distractions. The men, as already explained, only distracted subjects in a spatial-numerical sense. There was no evidence that the other objects actually distracted subjects as they attempted to solve the problem. Only 14 subjects used the loading ramp, often for loading only the first car or two. Nevertheless, manipulative solvers engaged in unnecessary simulations to a greater extent in these two problem settings than in the other three, as shown in Table 2. However, all 22 subjects involved were distracted by color-attribute or spatial-numerical cues.

Nothing external to the problem setting was considered to be a distraction for the purposes of this study. However, some subjects were possibly affected by extraneous perceptual cues. For example, 49 subjects looked at the tape-recorder. However, few did so more than once, and none seemed unduly concerned by it. Only five subjects actually commented on it. Thirty-five subjects were classified as being somewhat nervous in the problem-solving situation, including six in grade 3, thirteen in grade 2, and sixteen in grade 1. Nevertheless, all 360 subjects attempted the problem.

Of the 208 subjects distracted by spatial-numerical cues, 128 gave no explanation for their actions. They merely indicated

how they did it by alluding to the amount of room or number of spaces, rather than giving any reason why this strategy was used. In cases where these subjects were asked if they thought there were enough cars to fill each truck most answered in the affirmative. Sixty-one other subjects actually stated that they thought there were enough cars to fill every truck or, in the case of some manipulative solvers, that there were not enough cars to put the same number on each. For example, manipulative solvers who did not alter an unequal distribution of cars, such as 5,5,2, insisted that there were not enough cars to put the same number on every truck. Another seven subjects stated that they liked to or wanted to fill the trucks; four said the problem had to be done that way; and eight gave other reasons such as the need to take away as many cars as possible on each truck. In general, all the subjects distracted in this way tended to focus their attention on the trucks and ignore the fact that there were either 12 or 13 cars to distribute. However, some did overcome the distraction and proceed to solve the problem. That is, they attended to a spatial-numerical cue initially but then abandoned it as a means for solving the problem.

The men on the two trucks in the second and fifth problem settings acted as distractions for 37 subjects in a spatial-numerical sense because they took up space on the trucks and subjects did not consider moving them. Twenty-four subjects gave no reason for this, eight stated that they thought the men were not to be moved, three said the men had to stay to help load the cars, and two simply preferred the men to remain there. However, some of these who gave no

reason were not specifically asked why they would not move the men. Once again, some subjects overcame this distraction. Some of these went on to solve the problem, while others remained distracted by a spatial-numerical cue. For example, several subjects said there would be five cars on each truck because each man occupied a space, but then changed this response to six per truck.

Thirteen of the 42 subjects distracted by color stated that they wanted to match the colors of cars and trucks, but offered no other reasons for this behavior. Another 15 based their color match on aesthetics. These subjects stated that they wanted to match colors because this would look either "nice", "pretty", "good", "better", or "neat". Six subjects said they thought the colors had to be matched. The other eight subjects gave no reasons for basing arrangements of cars on color-attribute cues, but if the experimenter suggested the above reasons the subject would generally agree with one of them.

Subjects often overcame the color-attribute distraction, but some of these would then be distracted by the spatial-numerical cues. Thus, they would begin by matching colors then discard this approach and try to determine how many cars would fit on each truck. Fifty-seven of the 238 subjects who were actually distracted were able to overcome at least one distraction. The one problem setting showing a discrepancy was the third setting where 23 subjects overcame distractions -- chiefly the color-attribute cue. The frequencies of subjects who overcame distractions in each grade are shown in Table 7. Only 12 of the 86 subjects distracted in grade 1 succeeded

Table 7

Observed and Expected Frequencies of Distracted Subjects in Each Grade Who Succeeded or Failed in Overcoming Distractions

Grade	Overcame at Least One Distraction	Failed to Overcome Any Distractions	Total Distracted
1	12 (20.6)	74 (65.4)	86
2	19 (19.9)	64 (63.1)	83
3	26 (16.5)	43 (52.5)	69
1-3	57	181	238

in overcoming any distractions, while almost 38 percent of the distracted third-graders were successful in this regard. Overall, there was a highly significant difference between observed and expected frequencies in the three grade levels ($\chi^2 = 11.96$, $p < 0.005$).

Distractedness and Criterion Scores

Each subject could score zero, one, two, or three points for the problem-solving task. One point was allocated if the first response was correct, one point was allotted for the use of a valid process, and one point was awarded if the subject confirmed a correct response and demonstrated confidence in it. The decision on the final two points often had to be deferred until after the interview session. For example, a subject might state that he or she was certain there would be four cars on each truck. However,

after subsequent questioning it could turn out that this arrangement was based on a color-attribute or spatial-numerical cue, or that the subject was not really sure whether four was the correct number or not. Eighteen subjects overcame distractions and solved the problem during the interview session, although not specifically asked to do so. They seemed to notice their errors from the questions which were asked. However, their scores were not altered in any way as a result of this.

The frequency distribution of distracted and non-distracted subjects at each score level is shown in Table 8. The mean score for the total sample of 360 was 1.02. Less than the expected number of distracted subjects scored three, two, or one, while more than the expected number scored zero. Overall, the difference between the

Table 8

Observed and Expected Frequencies of Distracted and Non-Distracted Subjects at Each Score Level

Score	Distracted	Non-Distracted	Total
3	29 (58.8)	60 (30.2)	89
2	19 (25.1)	19 (12.9)	38
1	12 (15.2)	11 (7.8)	23
0	178 (138.8)	32 (71.2)	210
Total	238	122	360

observed and expected frequencies was highly significant ($\chi^2 = 109.5$, $p < 0.005$). Thus, distracted subjects scored much lower than non-distracted subjects. The mean scores for the two groups were 0.58 and 1.88 respectively, and the difference between these means was highly significant ($z = 9.69$, $p < 0.0002$).

Distractedness and Conceptual Tempo

The time which elapsed between the conclusion of the experimenter's repetition of the problem question and the subject's first solution was measured to the nearest second. Some subjects responded while the problem question was being repeated, and some responded immediately afterwards. The time recorded in these cases was zero seconds, and involved 83 subjects. Twenty-two subjects took longer than two minutes to supply a solution, and the maximum time taken was 440 seconds. The mean time taken was 38.7 seconds. The median time was approximately 11.5 seconds, since 179 subjects took 11 seconds or less and 181 took more than 11 seconds.

The 179 fast responders were classified as "impulsive" subjects and the 181 slow responders were classified as "reflective" subjects. The meanings of these two terms, as used here, are therefore somewhat different from those generally accepted in the literature (Kagan, 1965). Mean scores and standard deviations of groups based on the dimensions of conceptual tempo and distractedness are shown in Table 9. Reflective subjects scored much higher than impulsive subjects. The difference of 0.81 in mean scores was highly significant ($z = 6.27$, $p < 0.0002$). However, conceptual tempo was

Table 9

Mean Scores and Standard Deviations of Distracted, Non-Distracted, Impulsive and Reflective Groups

Conceptual Tempo	Distracted		Non-Distracted		All Subjects	
	$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S
Impulsive	0.21	0.63	1.50	1.35	0.61	1.09
Reflective	0.97	1.28	2.20	1.10	1.42	1.35
All Subjects	0.58	1.07	1.88	1.27	1.02	1.30

independent of distractedness. Reflective subjects were just as likely to be distracted as impulsive subjects. Of the 181 reflective subjects, 115 or 64 percent were distracted, and 123 or 69 percent of the 179 impulsive subjects were distracted. Conceptual tempo was also independent of sex, ability, and grade level.

The distracted impulsive group was the lowest scorer with a mean of 0.21. The distracted reflective group was more successful with a mean score of 0.97 ($z = 5.71$, $p < 0.0002$). However, this group did not score as high as the non-distracted impulsive group ($z = 2.47$, $p < 0.01$), which in turn did not perform as well as the non-distracted reflective group ($z = 3.09$, $p < 0.001$). Thus, impulsive subjects who were distracted had minimal success with the problem, whereas the reflective subjects who were not distracted were far more successful with a mean score of 2.20 ($z = 13.49$, $p < 0.0002$). The relationship between conceptual tempo and distractedness is illustrated in Figure 1. There was no interaction between these two factors.

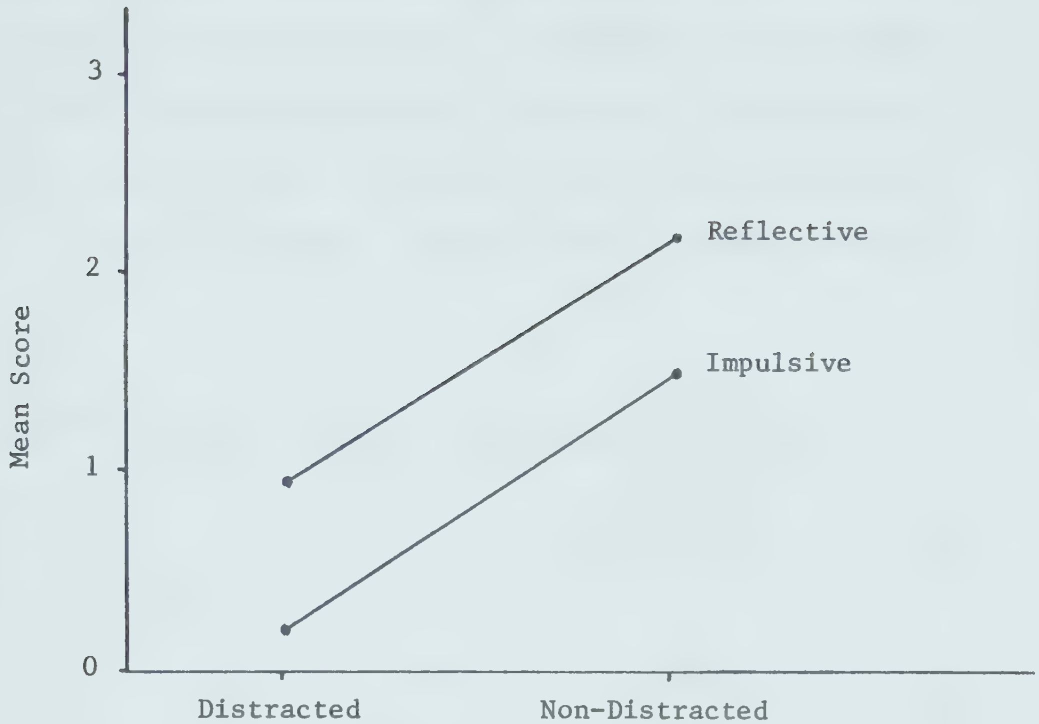

Figure 1

Relationship Between Distractedness and Conceptual Tempo

Relationships of Distractedness to Other Factors

Manipulative solvers were no more likely to be distracted than verbal solvers. Although 71 percent of the 116 manipulative solvers were distracted, compared with 64 percent of the verbal solvers the difference was not significant. Boys were distracted more than girls ($z = 2.00$, $p < 0.025$). The respective proportions distracted were 71 percent and 61 percent. Among manipulative solvers 71 percent of girls and 70 percent of boys were distracted. However, among verbal solvers 72 percent of boys but only 57 percent of girls were distracted. Therefore the difference due to sex was accounted

for by the fact that girls who used a verbal solution method were less inclined to be distracted than the boys who used this method.

The observed and expected frequencies of distracted and non-distracted subjects are shown in Table 10. The number of subjects distracted increased from 28 in the first problem setting

Table 10

Observed and Expected Frequencies of Distracted and Non-Distracted Subjects in Each Problem Setting

Problem Setting	Distracted	Non-Distracted	Total
1	28 (47.6)	44 (24.4)	72
2	36 (47.6)	36 (24.4)	72
3	46 (47.6)	26 (24.4)	72
4	64 (47.6)	8 (24.4)	72
5	64 (47.6)	8 (24.4)	72
1-5	238	122	360

to 64 in each of the fourth and fifth settings. More than the expected number of subjects were distracted in these two problem settings, while less than the expected number were distracted in the first three settings. Overall, distractedness was dependent on the problem setting to a highly significant extent ($\chi^2 = 49.0$, $p < 0.005$). This relationship became even more apparent when subjects who overcame distractions were taken into account. The frequencies of dis-

tracted subjects who failed to overcome any distractions were 20, 29, 23, 57, and 52 for the five respective settings. This re-emphasized the strong influence of distractions in the fourth and fifth problem settings.

The attention to distractions by subjects in the three grade levels is shown in Table 11. Eighty-six subjects were distracted in grade 1, 83 in grade 2, and 69 in grade 3. There was a more marked decrease in the number of distracted subjects from grade 2 to grade 3. Overall, grade level and distractedness were significantly related ($\chi^2 = 6.12$, $p < 0.05$). The higher the grade the less likely that subjects were distracted. Ability had some effect on distractedness. Over 69 percent of the low ability group was distracted, while approximately 62 percent of the high ability group attended to distractions. However, the difference between

Table 11

Observed and Expected Frequencies of Distracted and Non-Distracted Subjects at Each Grade Level

Grade	Distracted	Non-Distracted	Total
1	86 (79.3)	34 (40.7)	120
2	83 (79.3)	37 (40.7)	120
3	69 (79.3)	51 (40.7)	120
1-3	238	122	360

proportions distracted in each of the two ability levels was not great ($z = 1.34$, $p < 0.10$). In problem B 71 percent of subjects were distracted, whereas in the less difficult problem A 61 percent were distracted. Thus, subjects were more likely to be distracted in the division problem involving a remainder than when there was no remainder ($z = 2.00$, $p < 0.025$).

Results of Five-Way ANOVA

A five-way analysis of variance was carried out with sex, ability, grade, problem difficulty, and problem setting as categorical independent variables, and the criterion score for the problem-solving task as the dependent variable. The results are shown in Table 12. A number of significant and highly significant main effects and interaction effects may be noted. Although girls tended to score higher than boys there was no significant effect due to sex. However, all four other main effects were highly significant, as follows: ability ($F = 9.395$, $p < 0.00243$), grade level ($F = 18.874$, $p < 0.00000$), problem difficulty ($F = 30.862$, $p < 0.00000$), and problem setting ($F = 11.029$, $p < 0.00000$).

The interaction between problem difficulty and grade level was highly significant ($F = 4.833$, $p < 0.00876$). Two other interactions were significant. These were as follows: problem difficulty and ability ($F = 4.046$, $p < 0.04541$); and the interaction between sex, problem difficulty, ability and problem setting ($F = 3.185$, $p < 0.01422$). A further two cases of interaction were worth noting. The first was between problem difficulty, ability,

Table 12

Results of Five-Way ANOVA

Source*	Sum of Squares	df	Mean Square	F Ratio	Probability
A	2.844	1	2.844	2.349	0.12672
B	37.378	1	37.378	30.862	0.00000
AB	0.100	1	0.100	0.083	0.77410
C	11.378	1	11.378	9.395	0.00243
AC	1.878	1	1.878	1.550	0.21429
BC	4.900	1	4.900	4.046	0.04541
ABC	0.044	1	0.044	0.037	0.84827
D	45.717	2	22.858	18.874	0.00000
AD	5.906	2	2.953	2.438	0.08949
BD	11.706	2	5.853	4.836	0.00876
ABD	3.050	2	1.525	1.259	0.28576
CD	3.939	2	1.969	1.626	0.19895
ACD	2.839	2	1.419	1.172	0.31151
BCD	3.617	2	1.808	1.493	0.22675
ABCD	0.739	2	0.369	0.305	0.73737
E	53.428	4	13.357	11.029	0.00000
AE	8.072	4	2.018	1.666	0.15850
BE	6.872	4	1.718	1.419	0.22841
ABE	0.872	4	0.218	0.180	0.94858
CE	3.650	4	0.912	0.753	0.55658
ACE	4.206	4	1.051	0.868	0.48369
BCE	11.072	4	2.768	2.286	0.06086
ABCE	15.428	4	3.857	3.185	0.01422
DE	12.089	8	1.511	1.248	0.27204
ADE	5.344	8	0.668	0.552	0.81668
BDE	6.544	8	0.818	0.675	0.71297
ABDE	6.644	8	0.830	0.686	0.70394
CDE	11.700	8	1.462	1.208	0.29508
ACDE	10.578	8	1.322	1.092	0.36953
BCDE	5.244	8	0.656	0.541	0.82473
ABCDE	15.456	8	1.932	1.595	0.12683
Error	290.666	240	1.211		

*A = Sex, B = Difficulty, C = Ability, D = Grade, E = Setting

and problem setting ($F = 2.286$, $p < 0.06086$); and the second was between sex and grade level ($F = 2.438$, $p < 0.08949$). A more detailed analysis of main effects and interaction effects may be found in the sections which follow.

The Effect of Problem Setting and its Relationship to Other Factors

The problem settings, which reflected different types and levels of distraction had a very significant effect on scores, as shown by the results of the five-way analysis of variance. The mean scores of each grade for problem A in the five settings are given in Table 13. The studentized range statistic, or q test, was applied to test for differences between means within single categories (Winer, 1971, pp 195-187). The critical values of q were at either the 0.05 or the 0.01 levels of significance. There were no significant

Table 13

Mean Scores on Problem A for Each Grade in the Five Problem Settings

Grade	Problem Setting					
	1	2	3	4	5	1-5
1	0.67	1.00	1.17	0.25	0.00	0.62
2	2.08	2.17	1.58	0.67	1.00	1.50
3	2.25	2.50	2.17	0.92	1.67	1.90
1-3	1.67	1.89	1.64	0.61	0.89	1.34

differences between scores on problem A for grade 1, although none of the 12 subjects in the fifth setting scored any points at all. In grade 2 the group in the fourth problem setting scored lower than each of the groups in the first ($p < 0.05$) and second settings ($p < 0.01$). These same results also applied to grade 3. When all three grades were considered together, the groups in the first three settings each scored higher than the one in the fourth problem setting ($p < 0.01$). Also, the groups in the first and third settings each scored higher than the one in the fifth setting ($p < 0.05$), and so did the group in the second setting ($p < 0.01$).

The lowest mean score on problem A was in the fourth problem setting for both grade 3 and grade 2, while this was the second lowest for grade 1. On the whole, subjects had more success with problem A in the first three settings than in either the fourth or fifth settings. It should also be recalled that subjects were distracted less in the first three than in the last two problem settings. In the fourth setting subjects at all three grade levels had limited success with problem A. In both the third and fourth settings there were no significant differences between grades, although the trend was for scores to increase with grade. Both the grade 3 and grade 2 groups scored significantly higher than the grade 1 group in the first setting ($p < 0.01$). The same results applied in the second setting, except that the increase from first to second grade was not as great ($p < 0.05$). In the fifth problem setting third-graders scored much higher than first-graders ($p < 0.01$).

The mean scores on problem B for each grade in the five

problem settings are shown in Table 14. This division problem, which involved a remainder, proved to be far more difficult than problem A. As can be seen in Table 14, the mean scores were generally low for most groups. Thus, q tests revealed few significant differences between means. There were no significant differences between settings at any grade level. However, when all three grades were considered

Table 14

Mean Scores on Problem B for Each Grade in the Five Problem Settings

Grade	Problem Setting					
	1	2	3	4	5	1-5
1	0.25	0.67	0.58	0.67	0.17	0.47
2	1.08	0.58	1.33	0.08	0.50	0.72
3	1.33	1.17	1.33	0.33	0.33	0.90
1-3	0.89	0.81	1.08	0.36	0.33	0.69

together the group in the third setting scored higher than each of the groups in the fourth and fifth settings ($p < 0.05$). In the first setting the mean score for the grade 3 group was significantly higher than that for grade 1 ($p < 0.05$). However, there were no significant differences between grades in any of the other four problem settings.

As shown in Figure 2, mean scores for problem A were considerably higher than for problem B in all five settings. In each of the first two problem settings the difference was highly significant ($p < 0.01$) according to q tests. In the third and fifth settings

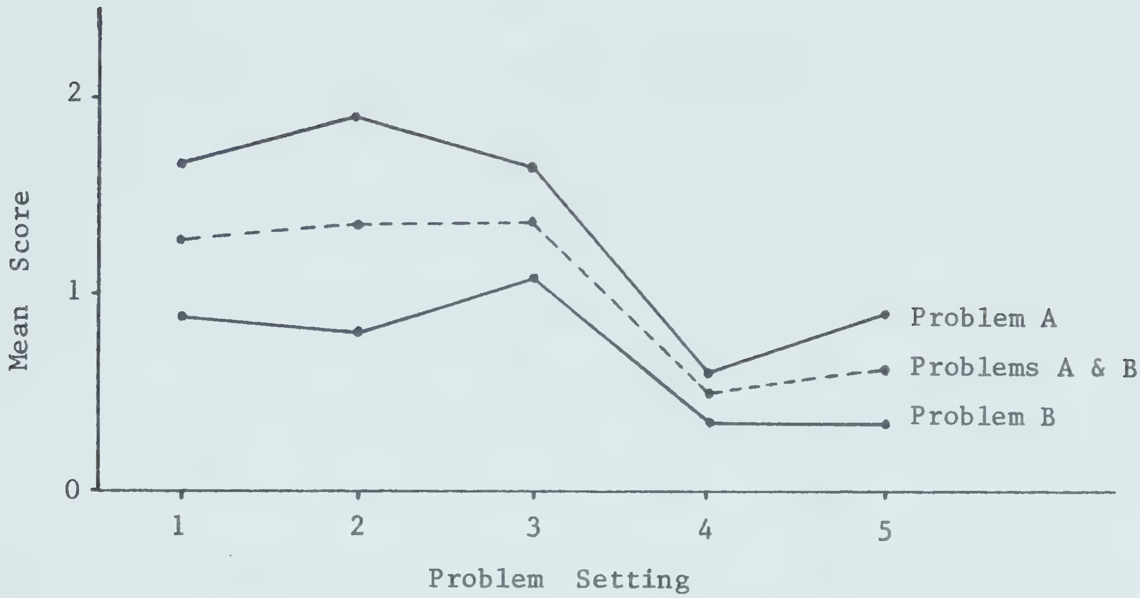

Figure 2

Relationship Between Problem Difficulty and Problem Setting

the differences were significant ($p < 0.05$), but not so in the fourth problem setting. In the fourth and fifth settings mean scores for problem A fell sharply to the general level of those for problem B in the first three settings. In considering both problems together, the mean scores in each of the first three settings were much higher than those in each of the other two settings ($p < 0.01$), as clearly illustrated in the graph.

The mean scores for problem A and problem B combined for each grade level in the five problem settings are shown in Table 15. Once again, q tests were used to check differences between means in each category. First-graders could not handle either problem very well and registered comparatively low scores in all five settings, so there were no significant differences. In grade 2 the mean for

Table 15

Mean Scores on Both Problems for Each Grade in the Five Problem Settings

Grade	Problem Setting					
	1	2	3	4	5	1-5
1	0.46	0.83	0.88	0.46	0.08	0.54
2	1.58	1.38	1.46	0.38	0.75	1.11
3	1.79	1.83	1.75	0.63	1.00	1.40
1-3	1.28	1.35	1.36	0.49	0.61	1.02

the fourth setting was lower than for the first and third settings ($p < 0.01$), and also lower than for the second problem setting ($p < 0.05$). Third-graders scored much lower in the fourth than in any of the first three settings ($p < 0.01$). There were no differences between scores in the first three settings for any grade. When all three grades were considered together there were also no differences between scores in the first three problem settings, as can be seen in Table 15. However, the scores registered in both the fourth and fifth problem settings were each much lower than scores in any of the other three settings ($p < 0.01$).

The relationship between grade level and problem setting is illustrated in Figure 3. The successive rise in score with grade level is evident in all problem settings except the fourth one. In the first setting both second- and third-graders scored higher than the grade 1 group ($p < 0.01$). In the second, third, and fifth set-

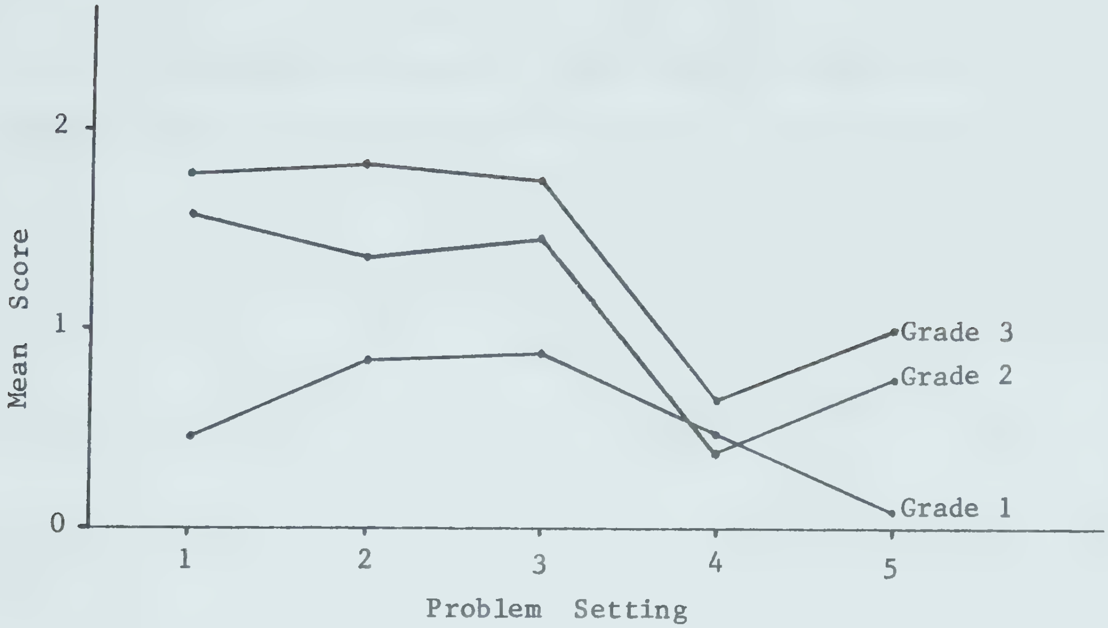

Figure 3

Relationship Between Grade and Problem Setting

tings third-graders scored significantly higher than first-graders ($p < 0.05$). The fourth problem setting included only the spatial-numerical distraction. However, the fifth setting contained this distraction together with the distractions included in the second and third settings. It was therefore reasonable to expect that, if distractions hampered success, scores would be lower in the fifth than in the fourth setting, but this was not generally the case. Although grade 1 scores fell in the fifth setting, second- and third- graders performed somewhat better in this setting than in the fourth problem setting.

The mean scores for boys and girls in each setting are given in Table 16, and the relationship between sex and problem setting is illustrated in Figure 4. It is clear from the graph

Table 16

Mean Scores for Each Sex in the Five Problem Settings

Sex	Problem Setting					
	1	2	3	4	5	1-5
Boys	1.31	1.44	1.08	0.44	0.36	0.93
Girls	1.25	1.25	1.64	0.53	0.86	1.11

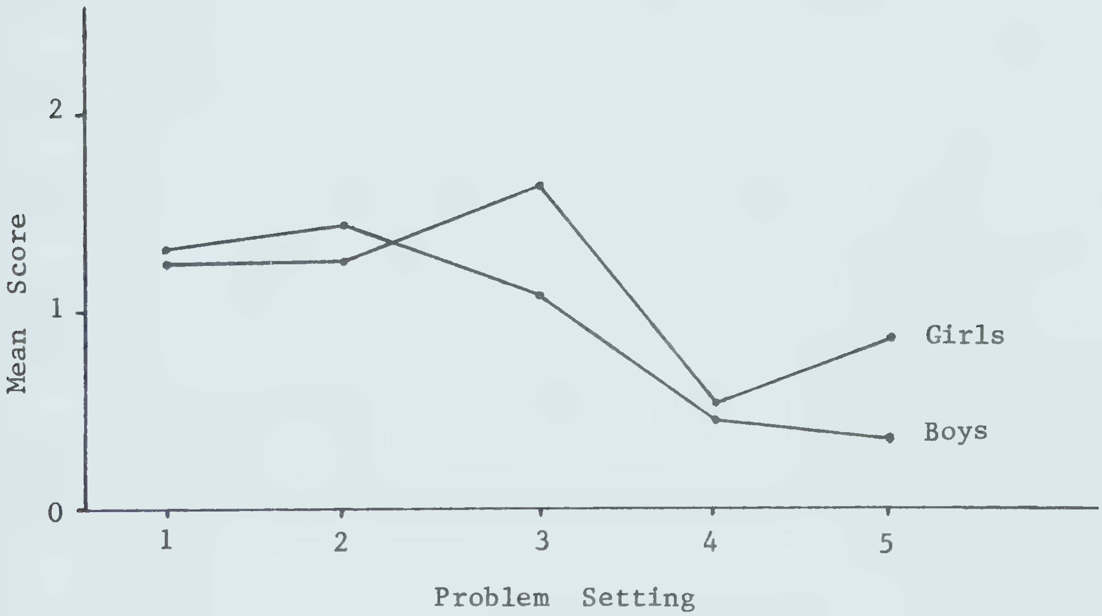

Figure 4

Relationship Between Sex and Problem Setting

that girls were superior to boys in the third and fifth settings, but differences were negligible in each of the remaining three problem settings. In the third and fifth settings q tests revealed significant differences between boys and girls ($p < 0.05$). The feature common to these two settings, but not occurring elsewhere,

was the three different colors of both cars and trucks. An examination of the distracted subjects in these two settings showed that 24 boys were distracted by a color-attribute cue, and only seven of these overcame the distraction. However, 18 girls were distracted by color and 12 of these succeeded in overcoming the distraction. Thus, girls tended to be more adept at dealing with color-attribute distractions than boys.

Mean scores for high and low ability groups in each problem setting are given in Table 17, and the relationship between ability and setting is illustrated in Figure 5. It is clear from the graph that the high ability group was superior to the low ability group in all cases except the fourth setting. However, q tests within individual settings showed that only in the third problem setting was the high ability group significantly superior ($p < 0.05$). Differences between means across the problem settings were more marked for the high ability subjects than for the low ability subjects, where there was only one significant difference;

Table 17

Mean Scores of High and Low Ability Groups in the Five Problem Settings

Ability	Problem Setting					
	1	2	3	4	5	1-5
High	1.50	1.53	1.64	0.47	0.83	1.19
Low	1.06	1.17	1.08	0.50	0.39	0.84

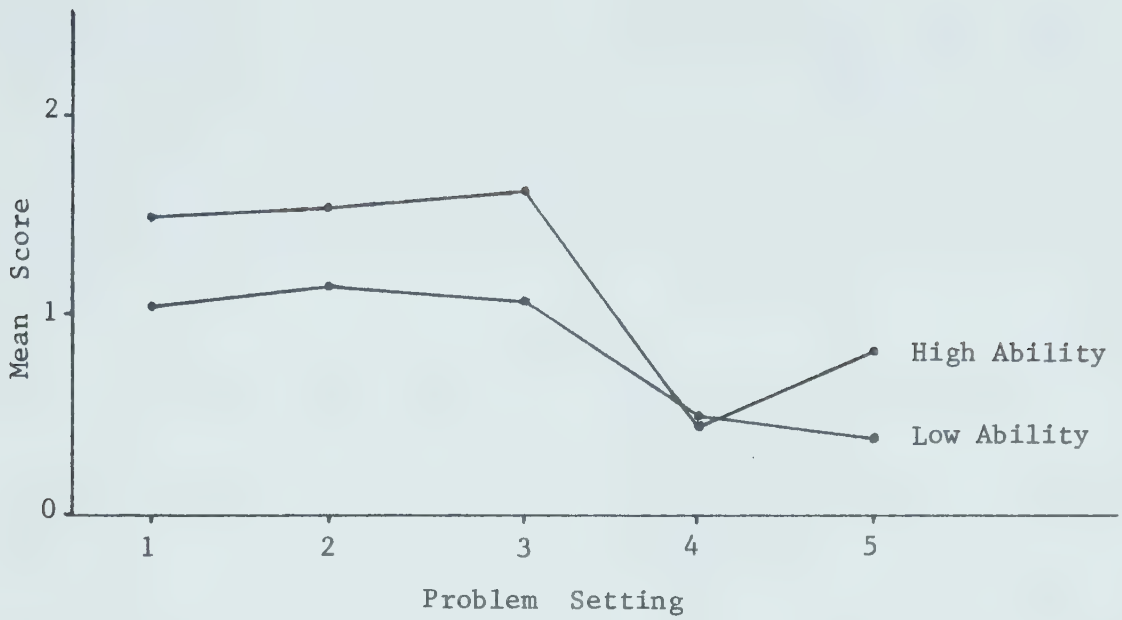

Figure 5

Relationship Between Ability and Problem Setting

namely, a higher mean score in the second than in the fifth setting ($p < 0.05$). The high ability group in the fourth setting scored much lower than those in each of the first three settings ($p < 0.01$). Also, the group in the third setting scored higher than that in the fifth setting ($p < 0.05$). Both the high and low ability groups had limited success with the problems in the fourth setting which included the one spatial-numerical distraction. However, in the fifth setting with more distractions the scores for the low ability subjects fell further, while those for the high ability subjects rose somewhat.

Interaction Effects on Scores

As previously reported, the effects of both problem

difficulty and ability on criterion scores were highly significant. Mean scores of the ability groups in each of the problem difficulty levels are shown in Table 18. There was significant interaction between ability and problem difficulty, as illustrated in Figure 6.

Table 18

Mean Scores of High and Low Ability Groups on the Two Problem Difficulty Levels

Ability	Problem Difficulty		
	A	B	A & B
High	1.40	0.99	1.19
Low	1.28	0.40	0.84
Both	1.34	0.69	1.02

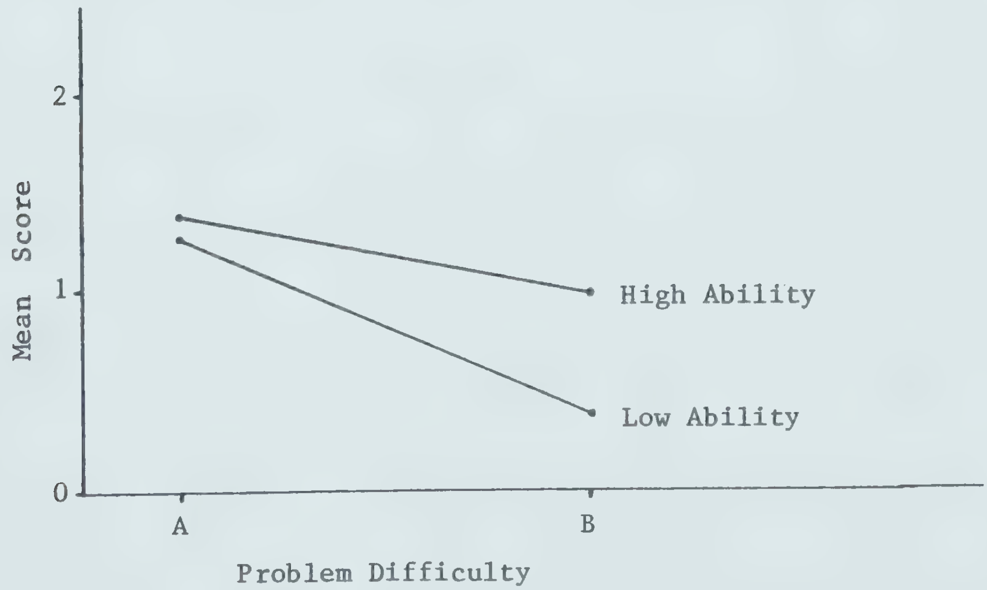

Figure 6

Interaction Between Ability and Problem Difficulty

In problem A both ability groups' scores were much the same, whereas in problem B the q test showed that the high ability group was far superior ($p < 0.01$). Thus, most of the difference in scores due to ability was accounted for in problem B. High ability students performed better in problem A than in problem B ($p < 0.05$), and the same was true of the low ability group except that the difference was greater ($p < 0.01$). Therefore, the majority of the difference in scores due to problem difficulty was accounted for by the low ability subjects.

The effect of grade level on the criterion score was highly significant as noted from the results of the five-way ANOVA. There was also a highly significant interaction between grade and problem difficulty. The mean scores for each grade in the problem difficulty levels are given in Table 19, and the interaction is apparent in Figure 7. Scores in problem B increased very slightly

Table 19

Mean Scores of Each Grade on the Two Problem Difficulty Levels

Problem Difficulty	Grade			
	1	2	3	1-3
A	0.62	1.50	1.90	1.34
B	0.47	0.72	0.90	0.69
A & B	0.54	1.11	1.40	1.02

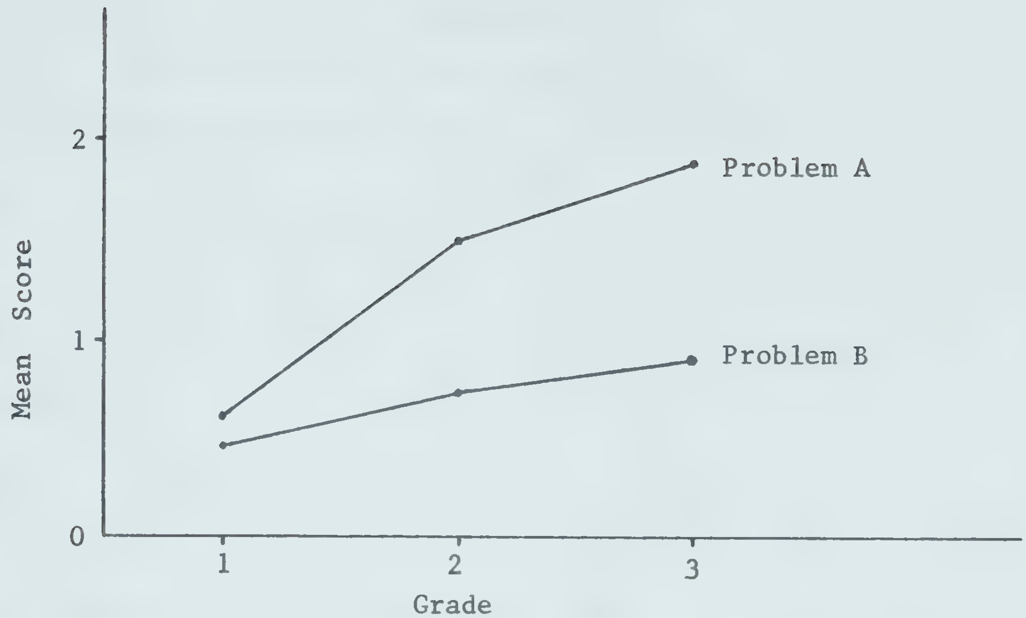

Figure 7

Interaction Between Grade and Problem Difficulty

from grade to grade and q tests revealed no significant differences between means. This proved to be a tough problem for all three grade levels. However, in problem A scores climbed more steeply through the three grades, with both second- and third-graders scoring much higher than first-graders ($p < 0.01$). Grade 1 subjects found both problems A and B to be difficult and the difference between scores was minimal. However, in each of the next two grades the difference was very marked ($p < 0.01$). Much of the overall effect on scores attributed to grade level resulted from the low mean score of grade 1 subjects (0.54) in comparison with the scores for second- and third-graders.

The mean scores of boys and girls in each grade are given in Table 20. The five-way ANOVA showed some interaction

Table 20

Mean Scores of Boys and Girls in Each Grade

Sex	Grade			
	1	2	3	1-3
Boys	0.63	0.92	1.23	0.93
Girls	0.45	1.30	1.57	1.11
Both	0.54	1.11	1.40	1.02

between grade and sex ($F = 2.438$, $p < 0.08949$), as illustrated in Figure 8. There was no significant difference between boys and girls at any grade level. Overall, girls tended to perform better than boys but the difference was not significant due to the

Figure 8

Interaction Between Sex and Grade

comparatively low score of the first-grade girls.

Two other interaction effects were noted in the results of the five-way ANOVA. There was interaction between ability, problem difficulty, and problem setting ($F = 2.286$, $p < 0.06086$). Finally, there was significant interaction between sex, ability, problem difficulty, and problem setting ($F = 3.185$, $p < 0.01422$).

Major Findings

The results of the investigation have been reported in this chapter. The major findings of the study were as follows.

1. Subjects exhibited a wide range of problem-solving behaviors which, in many cases, were influenced by the problem setting.
2. Nearly all the subjects used either an invalid process or a trial and error approach when attempting the problem.
3. A majority of the subjects were distracted by irrelevant spatial-numerical or color-attribute cues.
4. Distractedness was related to problem setting, and each of these factors affected problem-solving performance.
5. The ability to cope with distractions increased with the grade level.
6. Girls were better able to cope with distractions than boys.
7. High ability subjects were more successful than low ability subjects on the problem-solving task.
8. More subjects were distracted in the division problem with a remainder (problem B) than in the one with no remainder (problem A).

9. Slow (reflective) responders were more successful on the problem-solving task than fast (impulsive) responders, but distractedness was independent of conceptual tempo.
10. There were a number of interaction effects.
11. There were some defects in the interview techniques.

CHAPTER IV

SUMMARY, FINDINGS AND IMPLICATIONS, CONCLUSIONS, AND RECOMMENDATIONS

A summary of the investigation is presented in the first part of this chapter. The findings and their implications are then discussed in detail. The next section contains a list of conclusions drawn from the study. Finally, some recommendations are proposed for both research and teaching.

Summary

In the field of mathematics education more research has been directed at problem solving than at any other topic, but only a few of these investigations have been concerned with non-verbal problems. Some studies have shown that the inclusion of noise or distractions in word problems tends to increase problem difficulty. The Piagetian and related studies show that young children experience difficulties in developing specific concepts due to their attention to irrelevant perceptual cues in concrete and pictorial material. Although there is some evidence that young children are also distracted in non-verbal mathematical problems, no studies have systematically investigated the role of distractions in such problems.

The major purposes of this study were as follows:

(i) observe, record, and analyze behaviors of first-, second- and third-graders attempting to solve one of two non-verbal

partitive division problems (one with a remainder; the other with no remainder) in problem settings representing different qualities and quantities of distractions;

(ii) investigate subjects' interactions with distractions in the five problem settings;

(iii) examine the effects of distractedness and conceptual tempo on problem-solving achievement, and the relationships of these factors to sex, ability, grade level, problem difficulty, and problem setting; and

(iv) determine the main effects of sex, grade, ability, problem difficulty, and problem setting on problem-solving achievement, and also determine any interaction effects.

The data for this study were collected in November and December, 1976. Six schools were chosen from those within the Edmonton Public School system so as to represent, as far as practicable, the population of first-, second-, and third-graders in the system. Stratified random sampling procedures, based on sex and Metropolitan Readiness Test percentiles, were used to select ten matched groups from the grade 1 pupils in these schools. Each group consisted of three girls and three boys of high ability, and three girls and three boys of low ability. Ten matched groups were selected in the same way in each of the other two grades, except that percentiles on a standardized mathematics test of the Edmonton Public School Board were used as the criterion for ability.

At each grade level each group was randomly assigned to one of two non-verbal partitive division problems in one of five

distraction settings. Each setting included three trucks, and 12 cars for the problem with no remainder or 13 cars for the problem involving a remainder. In the first problem setting the cars and trucks were all the same color to create minimal distraction. This also applied in the second setting except that other objects were also included to add realism and present situational distractions. The third setting embodied color-attribute distractions in that the three trucks were each of a different color, and there were three colors of cars to match. All cars and trucks were of the same color in the fourth setting, but a spatial-numerical distraction was added by marking six bays on the back of each truck. The fifth problem setting contained all the distractions of the other settings in combination.

Each of the 360 subjects attempted one of the two problems individually with the experimenter. The subject was asked how many cars would be on each truck if as many as possible of the 12 (or 13) cars were loaded so as to put the same number on every truck. A standard format was adhered to in every case up to the point of the child's confirmation of his or her response. Thereafter the structure of the interview varied, depending on the subject's actions and responses. This latter part of the interview was designed to get at the thought processes used in the problem-solving task; in particular, whether or not the subject was distracted and the reasons for such distractions. An audio tape-recorder was used to record verbalizations, and all overt behaviors were noted on data cards. Each subject could score one point for each of the following: first response correct, use

of a valid process, and confirmation of the correct response. Thus, a score of either zero, one, two, or three was awarded.

The time taken for each subject to give the first response was noted. Subjects were classified as either fast responders (impulsive) or slow responders (reflective) for conceptual tempo. They were also classified as either verbal or manipulative solvers, based on whether or not they manipulated the materials when attempting to solve the problem. A five-way analysis of variance was carried out to determine the main effects of grade, sex, ability, problem difficulty, and problem setting on problem-solving performance, and also any interaction effects. Chi-square goodness-of-fit tests and tests for differences between proportions or means were used to investigate other relationships.

The subjects exhibited a wide range of problem-solving behaviors; many of which were influenced by the particular problem setting. Only 36 percent of subjects used a valid process in attempting to solve the problem, and most of these followed a trial and error approach. Very few distributed one car per truck in turn, or used computation to solve the problem. Nearly two-thirds of the subjects were distracted by irrelevant spatial-numerical or color-attribute cues. That is, they considered that the number of cars to be put on each truck was based on the amount of room, the number of marked bays, or a matching of cars and trucks by color. However, some overcame the distractions and proceeded to solve the problem. A majority of the distracted

subjects gave no explanation for their attention to the irrelevant perceptual cues.

Distractedness was related to sex, grade level, problem difficulty, and problem setting. More subjects were distracted in grade 1 than in either of the other two grades. Subjects were distracted more in the problem involving a remainder than in the one with no remainder. The fourth spatial-numerical distraction setting and the fifth maximum distraction setting each proved to be more distracting than any one of the other three problem settings. In the fifth problem setting with bays marked on the trucks more subjects were distracted by the spatial-numerical than by the color-attribute cue. However, in the third setting with no markings on the trucks the reverse was true. Girls seemed better able to cope with the color-attribute distraction than boys, and among the verbal solvers girls were less distracted than boys. Subjects who were not distracted were far more successful on the problem-solving task than those who were distracted, and reflective subjects attained much higher scores than impulsive subjects, but there was no interaction between distractedness and conceptual tempo.

Problem-solving scores were significantly affected by grade, ability, problem difficulty, and problem setting. Success on the problem-solving task increased with both grade and ability. Subjects were much more successful on the problem with no remainder than on the problem which involved a remainder. In fact, children in all three grade levels had limited success with the latter problem. Lower scores were registered in the fourth and fifth problem

settings, where more subjects were distracted, than in any of the other three settings. In addition to these main effects, interaction effects of the following sets of factors were noted: ability and problem difficulty; grade and problem difficulty; grade and sex; ability, problem difficulty, and problem setting; and sex, ability, problem difficulty, and problem setting. Finally, there was evidence of some flaws in the interview techniques.

Discussion and Implications of Findings

The major findings of the investigations were listed at the conclusion of Chapter III. A discussion of each of these follows.

Subjects exhibited a wide range of problem-solving behaviors which, in many cases, were influenced by the problem setting.

In the fourth and fifth problem settings, where six bays were marked on each truck, most subjects simply counted the bays to determine their solution. It seemed that the marked bays lured many children into a different interpretation of the problem. Apparently, the problem was seen as one requiring the number of cars which would fit on each truck. Each bay could take one car, so the children generally counted the bays to see how many cars would fit on each truck. Thus, for these children, counting the bays was a valid and efficient strategy for solving the problem as they saw it. The most common response in these two settings was six cars per truck, which provided further evidence to support the notion that these children

were solving a different problem from the one assigned.

A similar interpretation of the problem seemed to occur in the first three settings with no bays marked on the trucks. Many children set out to determine how many cars would fit on each truck, as if this was the problem to be solved. Some subjects counted imaginary spaces on the trucks, while others loaded cars to check. Responses varied from three to twenty cars per truck, with the most prevalent being three, four, five, or six cars per truck. Thus, in all five settings, children tended to misinterpret the assigned problem and try to find out how many cars would fit on each truck. However, this behavior was less common when the trucks were unmarked. Apparently, the marked trucks provided a stronger distraction than the unmarked ones.

Other misinterpretations of the problem occurred in the third and fifth settings when different colored cars and trucks were used. In these settings some children seemed to think that the task required them to put the blue cars on the blue truck, the yellow cars on the yellow truck, and the red cars on the red truck. Thus, the arrangements 3,4,5 for problem A and 3,4,6 for problem B were based on the respective numbers of blue, yellow, and red cars. Some subjects appeared to see the task as one requiring both the same color and the same number of cars for each truck. For example, seven children said there would be three cars on each truck because there were only three blue cars for the blue truck.

When children give an incorrect response to a problem this may not mean that they cannot solve that problem; rather, it

may be that the response results from the solution of a different problem altogether. During the interview session which followed the problem-solving task, eighteen children solved the problem without being specifically asked to do so. The questions posed by the experimenter apparently led them to identify the problem assigned originally, and they were able to solve it. If a child gives a correct response to a problem in one setting this does not guarantee that the same response would be given to that problem in a different setting. It appears that children should be exposed to the same problem contained in a variety of settings, along the lines of the multiple embodiment principle proposed by Dienes (1971).

The problem question began with the phrase "we have to load . . .", and therefore implied that loading the cars on the trucks would be acceptable behavior. Yet fewer than one-third of the subjects were manipulative solvers. Obviously, some subjects were able to solve the problem without manipulations, but many could not. A reluctance to touch any of the objects suggested that the manipulation of materials to solve problems was not the usual thing to do for many children. Perhaps children are not given much experience with manipulatives in the school mathematics program. It was somewhat surprising to find that there was no significant decrease in the number of manipulative solvers with the increase in grade level. One possible explanation is that older children may be more inclined to search for rules to follow for a given task. They might interpret "we have to load . . ." as a rule, and therefore tend to manipulate the cars. On the

other hand, younger children may be less influenced by this cue.

In the fourth and fifth problem settings where bays were marked on the trucks there were fewer manipulative solutions than in any of the other three settings with unmarked trucks. As already indicated, many children misinterpreted the problem and set out to determine how many cars would fit on each truck. Thus, in the fourth and fifth settings it was a relatively simple procedure to count the marked bays, and that is what they tended to do. However, in the other settings children were more inclined to load cars to see how many would fit on each truck.

Some manipulative solvers engaged in unnecessary simulations, such as driving trucks around or using the loading ramp, which were not central to the problem. Such simulations were more prevalent among boys than girls, probably because boys tend to play with cars and trucks more often than girls do. The added realism in the second and fifth problem settings, which included objects other than cars and trucks, seemed to elicit more unnecessary simulations than the other three settings. These actions almost disappeared by grade 3, and all the subjects involved had little or no success with the problem. Dienes (1963) suggests that "in many cases play is resorted to as a kind of defence mechanism when the cognitive going gets too tough for the learner (p. 47)." The findings seem to support Dienes' claim.

Nearly all the subjects used either an invalid process or a trial and error approach when attempting the problem.

Sixty-four percent of the subjects did not use any valid

process at all, 32 percent employed trial and error, and four percent used more efficient processes when attempting the problem. Most of those using an invalid process attended to either color-attribute or spatial-numerical cues, as indicated previously. Others employed a number of invalid strategies. Six children said there would be three cars on each truck because there were three trucks. These children apparently perceived a different problem from the one presented. They seemed to think that, for each truck, there had to be the same number of cars as trucks. Twelve other subjects said they guessed to obtain their solution. Possibly, in some of these cases, a more rational strategy was used, but this could not be verified. The so-called guess may have been an attempt to explain how the problem was thought out. Thirteen other subjects tended to respond rather promptly, but could not provide any explanation of the strategy employed. Some possible reasons for this are discussed subsequently.

Of the 130 subjects who used a valid process, all but 15 employed a trial and error method. Eleven of the 15 subjects distributed one car per truck in rotation, while the remaining four subjects computed the solution by using the operations of multiplication or division. It was surprising that so few subjects used a systematic process. In the trial and error method several arrangements of cars would be tried in order to arrive at the solution. Some subjects solved the problem with only one trial. When asked why they chose four cars per truck to begin with, a few children made a response like "I just tried four and it worked". However,

many of these gave their solution with little or no hesitation but could not explain the initial choice of four, even though they usually demonstrated confidence in this solution.

Apparently, young children have considerable difficulty explaining their thought processes -- particularly when a problem is solved quickly. Menchinskaya (1969) claims that "when there is such an automatized solution, the subject often experiences great difficulty in explaining his thoughts, so rapidly do they proceed (p. 41)." Donaldson (1963) also contends that "much of the thought may be too quick for articulated speech or for some other reason quite inaccessible to consciousness (p. 29)." This statement agrees with Piaget's (1973) claim that "a larger part of the structures the child uses when he sets out actively to solve a problem remain unconscious (p. 86)." According to Freudenthal (1973), adults generally use an algorithm whereas young children tend to calculate more visually and are therefore unable to describe the process (p. 129). First-graders seemed to experience the most difficulty in explaining what they did. One explanation could be that young children generally solve problems at the perceptual or figurative levels, rather than at the operational level of functioning. Therefore, they may not have developed a schema for conceptualizing or describing the process.

As far as could be ascertained, only four of the 360 subjects used the operations of multiplication or division to solve the problem. This was a surprising result; particularly since instruction in these operations is a component of the school program

in grade 2. It was expected that many more subjects would solve the problem with no remainder, especially among the third-graders. Children in grade 3 are generally expected to be able to compute the number of threes in twelve, but it seems that they are unable to translate a real problem into this computational form; perhaps due to a lack of appropriate experiences. Eleven subjects solved the problem by distributing one car per truck in rotation. This partitioning process is the most efficient one for solving a partitive division problem in the enactive mode. It appears to be a relatively simple procedure for young children to learn, but the fact that so few children used this process suggests that they are not being given experience with it in school. Teachers should guide young children in partitioning a given set of objects into equivalent subsets by distributing the objects on a one-by-one basis. This procedure should be mastered before being translated into the operation of division at the symbolic level.

The overall performance of the subjects on the problems was somewhat low, and the lack of systematic procedures was very marked. Could it be that partitive division is rather difficult for children in these grades? Bourgeois (1976) found that young children were less successful with partitive than with measurement division problems, and earlier research has come up with a similar finding (Callahan & Glennon, 1975). However, the difference could well be due to the lack of attention being given to partitive division in school programs.

A majority of the subjects were distracted by irrelevant spatial-numerical or color-attribute cues.

The proportion of subjects distracted was surprisingly high. In all, 66 percent attended to distractions in the problem settings. Of these, 208 subjects were distracted by spatial-numerical cues, 42 were distracted by the colors of the cars and trucks, and one subject tried to classify cars by kind. A further nine percent of the subjects said they were distracted when questioned by the experimenter. However, the particular questions were such that they may have influenced the subjects' responses. Therefore, there was no way of telling whether or not these children were actually distracted.

As already indicated, the children who focused on irrelevant information seemed to identify a different problem from the one assigned. Apparently, for the distracted subjects the relevant information became irrelevant. For example, most of the subjects who were distracted by a spatial-numerical cue focused on the trucks to determine how many cars would fit, and completely ignored the fact that there were only 12 or 13 cars. To them, this information was irrelevant. The majority of the distracted subjects gave no explanation for their attention to the irrelevant cues. When questioned they often did not answer or said, "I don't know". Asking a subject distracted by a spatial-numerical cue the question "Why do you fill each truck?" is probably equivalent to asking the successful solver "Why do you put four on each truck?" Each subject might see the question as "Why did you solve the problem?" and consider it redundant, or be unsure how to respond. Also, some children may have been

unable to recall the rationale for the particular strategy employed.

There was no significant difference between the proportions of manipulative and verbal solvers who were distracted. However, in the pilot study (Bana & Nelson, in press) it was found that manipulative solvers were more easily distracted than verbal solvers, and also that the overall percentage of distracted subjects was less than in the current study. In the main study, subjects were questioned more extensively in an endeavor to determine whether they were distracted or not. Also, the SES of the pilot sample was higher than that of the main sample. Both of these factors may have contributed to the difference. The effect of SES on distractedness needs further investigation.

The problems used in this study had two components -- the problem setting, and the problem question presented verbally to the subject. Therefore, if a subject was distracted by irrelevancies in the problem setting the nature of the question probably played a key role also. Almy (1966) claims that

> the child's responses are shaped not so much by the question put to him, as by his way of looking at the materials and objects in the experiment (p. 133).

This may be so, but Gagné (1966) suggests that

> the process of distinguishing between relevant and irrelevant cues in the stimulus situation may be affected by instructions which increase the distinctiveness of these cues (p. 142).

In the current study "the same number of cars" was a key phrase in the problem question. Yet, subjects rarely used this phrase either when seeking clarification of the problem or during the interview which followed; rather, they tended to say "the same amount of cars".

If "same amount" was more meaningful than "same number" then the substitution of the latter phrase in the problem question might have resulted in fewer subjects being distracted. The overall mean score in the main study was 1.02, whereas the mean score in the pilot study was 1.58 (Bana & Nelson, in press). Some of the difference is almost certainly due to SES and related factors. However, the problem questions were not identical in the two studies, and this may also account for some of the discrepancy in performance. More research is needed to determine how the problem question affects distractedness in the problem setting.

The first problem setting was devised as one of minimal distraction but many subjects were distracted by a spatial-numerical cue, both in this and in the other four settings. The second and fifth problem settings included situational distractions in the form of three men, a building, a loading ramp, a parking area, and a roadway. However, these situational distractions apparently had no effect as such. Some subjects were distracted by the man on each of the first two trucks, but only in a spatial-numerical sense in that each man was considered to take up space. For example, these children generally claimed that each truck could take five cars because the man was in the way. Although these two problem settings elicited more unnecessary simulations than other settings, all the subjects who engaged in this behavior were distracted by spatial-numerical or color-attribute cues. Thus, it appears that a distraction only 'works' if it provides the basis for some alternative problem.

In devising problems at the concrete level for young

children it would appear that some difficulty will be encountered, both in isolating particular distractions and in providing situations which are distraction-free. Perhaps it is not possible to provide problem situations which are realistic and interesting, and at the same time free of distractions. As Stevenson (1975) observes, in making a problem interesting for the child we also introduce irrelevancies. How can this situation be reconciled? Dienes (1963) suggests that "perhaps the solution lies in gradually increasing or decreasing the amount of noise present (p. 56)." Dienes maintains that cutting through noise is an essential component of concept formation. Skemp (1971) agrees, and proposes the following:

> in the early stages, low noise -- clear embodiment of the concept, with little distracting detail -- is desirable; but as the concept becomes more strongly established, increasing noise teaches the recipient to abstract the conceptual properties from more difficult examples (p. 33).

If children are introduced to a problem containing many distractions the noise level may be such that they find the problem impossible to solve. In this respect, Skemp's proposition seems reasonable. Also, if the ability to cope with distractions is a developmental characteristic of children then minimal distractions in the early stages would be preferable. For young children, a gradual increase in the amount of noise may be the best approach. However, at some stage it might be better to introduce problem situations with many irrelevancies then gradually reduce the irrelevant information if necessary. In order to be successful at solving real problems children will need to develop the ability to handle

situations with high noise levels. Three possible solutions have been discussed: decreasing the noise level; increasing the noise level; or increasing the noise level for young children, then reversing this procedure later. The last one appears to be the best of these, but longitudinal research needs to be carried out to get more definitive answers. Teachers need to plan learning experiences carefully when using manipulative material with young children. They should be aware of attributes which may distract children.

Distractedness was related to problem setting, and each of these factors affected problem-solving performance.

The mean scores for distracted and non-distracted subjects were 0.58 and 1.88 respectively. The marked difference in scores is not surprising if one considers that a distracted subject may be trying to solve a different problem from the one presented. Some distracted subjects overcame the distraction and went on to solve the problem presented to them. Why this occurred with some children and not with others is unclear. Perhaps some discovered that the problem they had identified could not be solved. For instance, if the problem was seen as requiring every truck to be filled with the given cars then this was impossible. In fact, some children tried to do this and, when they ran out of cars, realized it could not be done so they proceeded to attempt the assigned problem. Thus, if a problem is found to be illogical the child may search for another problem to solve.

The numbers of subjects distracted in the five respective problem settings were 28, 36, 46, 64, and 64. Since distractedness was closely linked to performance, subjects were much less successful in the fourth and fifth problem settings than in any of the other three settings. The one spatial-numerical distraction in the fourth setting proved to be particularly strong. It seems that the marked bays highlighted a plausible alternative problem which could be identified readily by the children. Surprisingly, there was no increase in the number of distracted subjects in the fifth setting, which included all the distractions devised for the other four settings. In fact, performance here was slightly better than in the fourth setting. Perhaps the fifth problem setting, with its extra information load, necessitated some processing of the information to seek out a problem; unlike the fourth setting where, at least according to the distracted subjects, the problem was more obvious.

The third problem setting was designed to include a color-attribute distraction. However, a spatial-numerical distraction also entered into this setting. Color was the stronger of the two distractions here but the reverse was true in the fifth setting, probably because the spatial-numerical distraction was quite definitive in the form of the marked bays. Bourne (1966) has shown that both the "form and amount of redundancy are important determiners of performance in concept identification problems (p. 59)." The findings of the current study indicate that different irrelevancies do create different levels of distraction. As a result of number conservation tasks given to kindergarten children, Scandura and McGee (1972) concluded that "rela-

tively minor variations in the materials used may significantly affect the child's performance on a task (p. 344)."

A similar conclusion has been reached regarding Piagetian concepts. For example, Suydam (1976) points out that "unless the same questions and materials are used, results may differ (p. 103)." Piaget's experiments with conservation tend to be task-specific. However, similar research has shown that changes in the task variables can produce different results (Callahan & Glennon, 1975, p. 32). Thus, whether children are classified as conservers or non-conservers may depend on how they cope with distractions in the given task. Obviously, it is not possible to investigate the effects of distractions in every conceivable setting for any given problem, and this poses difficult questions related to curriculum design. For example, how can appropriate learning materials and activities be selected if little is known about the various inherent distractions? Much more research is needed to establish general patterns of distractedness in young children, in order that curriculum builders and teachers may be able to anticipate at least some of the distractions which are likely to inhibit the learner in particular contexts.

The ability to cope with distractions increased with the grade level.

The numbers of distracted subjects in grades 1, 2, and 3 were 86, 83, and 69 respectively. Of these, the corresponding numbers of subjects who overcame at least one distraction were 12, 19, and 26. Thus, with the increase in grade fewer subjects were

distracted, and an increasing proportion of distracted subjects were able to discard irrelevant cues. The scores on the problem-solving task increased markedly from grade to grade. As Shulman (1976) suggests, these developmental differences in performance on the same cognitive task seem to be due to the ability to cope with noise (p. 77). This developmental trend may be observed in the way children acquire Piagetian concepts. For example, children who can conserve numerousness have, over a period of time, learned to ignore irrelevant perceptual cues such as length or configuration.

Girls were better able to cope with distractions than boys.

More boys were distracted than girls overall, although this difference was actually confined to the verbal solvers. Girls performed better than boys in the third and fifth problem settings where there was a color-attribute distraction. Fewer girls were distracted by color, and they were more likely to discard this cue than the boys. This could be due to different experiences that boys and girls have had with this type of situation. The color-attribute distraction needs to be embodied in a variety of situations in order to ascertain the extent of sex differences in relation to this factor.

High ability subjects were more successful than low ability subjects on the problem-solving task.

The mean scores of the high and low ability groups were 1.19 and 0.84 respectively, and the corresponding percentages of

distracted subjects were 62 and 69 percent. Statistically, the effect of ability on performance was highly significant. However, in practical terms, the difference between the two groups was not as great as might be expected. It seems that the criteria used for ability in this study may not constitute the best predictor of success. Perhaps IQ would be a better predictor than ability as it was defined here. The relationship between distractedness and performance has already been discussed. The high ability group tended to be distracted less than the low ability group, but the difference was not very marked.

More subjects were distracted in the division problem with a remainder (problem B) than in the one with no remainder (problem A).

The mean score on problem A was 1.34, and on problem B it was 0.69. The percentages of distracted subjects for the two problems were 61 and 71 percent respectively. The difference in scores was much more marked than the difference in distractedness, as indicated by the respective levels of significance ($p < 0.00000$; $p < 0.025$). This appears to weaken the argument that performance -- that is, problem difficulty -- is closely linked to the distractions which are present. However, problem B included an extra car in each setting. Although this was not considered as a distraction for the purposes of the study, it seems to have acted as such. Children often seemed to think that the problem was to load all 13 cars and put the same number on each truck. Thus, as with other distracted subjects, they essentially identified a different problem from the

one presented. Some eventually realized that it was impossible to load all the cars and that one had to be left. Others seemed to see that the problem they were attempting could not be resolved, but they did not identify what the assigned problem required.

Slow (reflective) responders were more successful on the problem-solving task than fast (impulsive) responders, but distractedness was independent of conceptual tempo.

The respective mean scores of reflective and impulsive subjects were 1.42 and 0.61. The difference in scores was surprisingly large, but the superiority of the reflective subjects was anticipated from previous research (Cathcart & Liedtke, 1969). Although the proportion of distracted subjects tended to be higher in the impulsive than in the reflective group the difference was not significant. This seems to contradict the distractedness-performance link discussed before. But conceptual tempo was defined only in terms of the median time taken to provide the first response to the problem question. On this basis, all subjects who took more than 11 seconds were classified as reflective. Nearly one-fourth of the distracted subjects overcame at least one distraction, and most of these were in the reflective group. This was probably due to the extra time involved in discarding a distraction and seeking out an alternative problem. Therefore, although impulsive and reflective subjects were almost equally distracted, reflective subjects often overcame the distraction and went on to solve the given problem.

In the reflective group the subjects who were not distracted performed significantly better than those who were. The same

result occurred in the impulsive group, but there was no interaction at all between conceptual tempo and distractedness. Conceptual tempo was independent of grade, sex, or ability. It would be interesting to know whether the findings would have differed to any extent if Kagan's (1965) criteria had been used for conceptual tempo. The relationship of distractedness to other dimensions of learning style, such as field dependence-independence, should also be investigated. The marked difference in performance between reflective and impulsive subjects emphasizes the importance of conceptual tempo in the teaching and learning of mathematics. Teachers should allow children ample time to process the information, rather than hurry their response. In fact, as Schwebel and Schwebel (1974) have shown, it is probably best to have children delay their response for a certain period of time so as to encourage them to reflect on the problem.

There were a number of interaction effects.

There was a significant interaction between ability and problem difficulty, and between grade and problem difficulty. The performances of the high and low ability groups in problem A (no remainder) were almost the same. However, in problem B there was a marked difference in scores. While high ability children still had some success, the low ability group performed very poorly with the more difficult problem. First-graders had little success with either problem but, for problem A, scores rose sharply in grade 2 and were higher again in grade 3. However, the rise in scores through the grades was much slighter for problem B. These develop-

mental trends suggest that, in general, division problems are not very appropriate for grade 1, but are suitable for grades 2 and 3. However, even in these two grades, problems involving a remainder may be too difficult for children of lower ability.

There was some interaction between sex and grade level. In grade 1 the boys performed better than the girls, but in the other two grades the reverse was true. Although girls tended to score higher than boys overall, the difference was not significant. If first-grade boys have had more experience with toy cars and trucks than girls, this could explain the difference in performance in grade 1. Yet it contradicts the findings in the other two grades. One could speculate that perhaps girls at this level benefit more from schooling than do boys, and therefore perform somewhat better than boys at the second- and third-grade levels.

There were some defects in the interview techniques.

Some instances of induced distraction resulted from the interviews. These occurred when the subject was asked directly whether or not a particular distraction had some bearing on the strategy used to solve the problem. The experimenter was inclined to ask such a leading question when the information was not volunteered by the subject. When asked if they attended to a particular distraction many subjects replied in the affirmative, perhaps to please the experimenter. Any information obtained in this way is of no practical use since it cannot be interpreted. It would be more useful to know that a subject declined to answer a question than to

obtain a response suggested by someone else. When the objective is to determine the thought processes of the child, great care should be taken to avoid the inclusion of any clues in the questions being asked.

In this study subjects were generally asked to recall all the arrangements of cars they had tried in attempting to solve the problem, and their reason for attending to a distraction when this occurred. However, one or both of these questions were inadvertently omitted in some instances, with the consequent loss of useful information. The nature of an interview will depend largely on its major purpose; in particular, whether the objective is to gather quantitative or qualitative data. Flexibility is generally desirable so that questions can be varied according to a subject's actions and responses. If quantitative data is required then the interview necessitates some structure. For example, if the intention is to determine how many children counted the objects before solving the problem then the appropriate question would need to be asked in every case. Finally, the use of non-verbal problems combined with individual interviews seems to be the most useful means for gathering information on how young children learn mathematics.

Conclusions

The findings above are not all definitive. Some will need to be checked by further research. However, the following general concluding statements seem to be supported, in varying degrees, by the results of this study.

1. Children in grades 1-3 lack experience with realistic partitive division problems in the enactive mode, and tend to use inefficient or incorrect methods in attempting to solve such problems.

2. Both problem-solving behavior and performance are affected by the setting in which the problem is embodied.

3. Young children are easily distracted by irrelevant perceptual cues in the problem setting.

4. Distracted children seem to identify and solve a different problem from the one presented, unless they overcome distractions and recognize the assigned problem.

5. Irrelevant information in a problem setting acts as a distraction only if it can be incorporated into a plausible alternative problem by the child.

6. Developmental differences in problem-solving performance are related to the ability to cope with noise.

7. Girls are better able to cope with distractions than boys in grades 1-3.

8. Slow (reflective) responders are more successful problem solvers than fast (impulsive) responders.

9. Division problems with no remainders are appropriate for grades 2 and 3, but problems involving remainders are suitable only for the higher ability pupils in these grades.

10. The use of non-verbal problems, combined with individual interviews, provides a sound basis for investigating how young children learn mathematics.

Recommendations for Research and Teaching

One of the topics most in need of further research in this area is the combined effect of both verbal and perceptual irrelevancies on problem-solving behavior and performance. In this study the verbal problem question was not varied. Other studies have shown that variations in the format of a verbal problem do affect problem difficulty. The interaction between the verbal question and the problem setting needs to be investigated. This can be done by incorporating different types and amounts of distraction in both the verbal and perceptual components of the problem. Another question requiring some definitive answers concerns the way in which noise should be included in learning situations. Should distractions be gradually increased, or decreased? Or, would some combination of these two approaches provide the best learning sequence? It has been suggested that young children may need to learn mathematics through a gradual increase in the amount of irrelevant information but that at some later stage it might be preferable to present problems with many distractions which can be gradually reduced if necessary. However, much more evidence is required to determine which is the best path to follow.

The study undertaken here was essentially of an exploratory nature and therefore limited in scope. Each subject attempted only one problem. It would be interesting to observe the behaviors of individuals in a number of similar problems to check for any variation in distractedness with practice. The findings have rev-

ealed useful information on the role of distractions in particular partitive division problems. Much more research needs to be undertaken with many different problems in a variety of settings. It is likely that the mathematical ideas in a problem, the type of situational referent, and the particular setting for both of these, all affect the way in which children will handle any distractions that are present. The many different types of possible distractions need to be identified, and their effects measured -- for example, attributes such as size, kind, and the like.

The findings suggest that girls are less distracted, and seem better able to cope with color-attribute distractions than boys. To what extent do these differences apply? Obviously, this question can only be answered through further research with a variety of problems and settings. A comparison of the results of the pilot study and the main study suggest that SES should be considered as a possible predictor of distractedness. The role of other pupil characteristics, such as IQ, should also be investigated.

The reluctance of many subjects to manipulate the materials suggests that children in the junior elementary grades may not be given sufficient experiences with concrete material to help them learn mathematics. Very few subjects used an efficient strategy to solve the problem. Partitioning a set by using a one-by-one distribution of objects is a strategy which children in grades 2 and 3 should be able to master readily. Teachers should ensure that children are given adequate experience and practice with this method so as to enable them to handle partitive division problems in the enactive mode.

Teachers need to be aware that realistic settings will include irrelevancies which distract young children. However, for any given situation it may be difficult to predict which aspects will distract children. Nevertheless, some attempt should be made to identify likely distractions in learning activities involving manipulative materials. Then children will need to be observed closely to see how they handle these distractions. If a child fails to solve a problem this may not signify that he or she is unable to handle the mathematics involved. In such cases irrelevancies should be removed as far as practicable, since they may be too distracting. On the other hand, if a child succeeds with a problem in a particular setting teachers should not be deluded into thinking that this type of problem has been mastered. The child's performance should be checked in other settings also.

Impulsive responders are not as successful as reflective responders in problem-solving tasks. Therefore, teachers would be advised to delay children's responses and encourage them to reflect on the problem before giving their solution. This may provide an opportunity to overcome distractions. Teachers should also employ the multiple embodiment principle; that is, expose children to the same problem in a variety of contexts to help them deal with varying types and amounts of distraction. Children need to be able to cut through noise in order to abstract mathematics from their environment.

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APPENDICES

APPENDIX A

SAMPLE OF INTERVIEW TRANSCRIPTS

The following sample of thirty transcripts of interviews consists of one example drawn from each of the two problem difficulty levels, for each of the three grade levels, in each of the five problem settings. Each transcript begins at the point where the problem question has been asked for the second time in accordance with the set protocol. The verbalizations of the experimenter (E) are presented in standard script, while those of the subject (S) are in italic script. Non-verbal behaviors are recorded in parentheses.

First Problem Setting

Problem A

Jacqueline -- Grade 1: (S looks at trucks and responds immediately). *Nine.* Are you sure that's right? (She nods). Quite sure? *Yeah, because I think three goes here* (pointing to front of first truck), *three there* (pointing to middle of truck), *and then three more there* (pointing to back of truck). I see. Now are there enough cars to put nine on each truck? (She looks intently at cars, apparently counting, then shakes her head). So there aren't enough. Did you count the cars before to see if there were enough? *No, I just did that.* Did you think there were enough before to put nine

on every truck? *Well, there's three here* (indicating three cars); *as much as I said on the truck.* But are there enough to put nine on that truck, nine on that truck, and nine on that truck (indicating each truck in turn)? *No, only on that truck* (indicating first truck). Will that give us the same number of cars on every truck? *No, because that one will get zero, that one will get zero, and that one will get nine.* Why did you want to do that? *Because, you see, there's three* (indicating first three cars) *and that would fit there* (pointing to front truck), *and that three would fit there* (indicating a second group of three for the middle of the truck), *and three there* (indicating a third group of three for the back of the truck); *because it looks like it fits.* I see, but why did you want to put that many? *I dunno.* Did you want to fill it up? *Yeah, to take them away.* Did you think there would be enough cars to put nine on each truck? Did you think that before, did you? *I didn't quite know, then. There's only three here* (pointing to groups of cars); *here's three cars, there's three cars, there's three cars, and there's three cars. There's four. One, two, three, four* (counting groups of three). But does that give us the same numer of cars on every truck? *No, zero there, three there, and nine there* (indicating each truck). So, that's no good, eh? *Uh-uh.*

Adrienne -- Grade 2: (S looks at cars for some time; seems to be counting). *Four.* Are you sure that's right? (She nods). Are you quite sure? *Uh-huh.* How did you know there would be four on each truck? *I counted.* Show me how you counted. (She points as she counts groups of four cars). *One, two, three, four; one, two, three, four;*

one, two, three, four. I see, but how did you know you had to count to four? Did you try counting to three? *Uh-huh.* What happened when you tried to count to three? *There were some more left.* What did you try first? *Two.* You counted in twos, did you? *I counted in twos, then threes, then fours.* How do you know four is right? *Because I counted three groups.*

Shelley -- Grade 3: (S looks at cars, counting to self). *Four.* Are you sure that's right? *Uh-huh.* Quite sure? *Uh-huh.* How did you know there would be four on each truck? *Because there's twelve cars and you put four on each one, and that would work because four times . . . Oh, what am I doing? I can't explain it.* You try. *Well, there's twelve cars and you can put four on each truck, so, umm . . .* Did you try any other number first? *Yeah.* What did you try first? *I tried six, but it didn't work.* What made you try six? *Because then you could only fill up two trucks.* What made you try six? *Because six plus six equals twelve. But then you can't do it because you only have two trucks.* Did you decide to try six because you could only fit six on the back of the truck, or what? *Yeah.* Why did you want to fill the truck up? *I dunno.* Did you think there would be enough cars to put six on each? *Yeah.* Did you count all the cars to see if there would be enough? *Yeah.* And when six wouldn't work, what did you do then? *I tried other numbers.* What did you try? *I tried three.* Did that work? *No.* What did you try next? *Then I tried two, but that didn't work either.* What did you do then? *So, I tried four, and that worked.* Is there any other way you could figure it out? *Er . . . I don't think so.*

Problem B.

Robert -- Grade 1: (S looks at the cars, then at trucks). *Nine.* Are you sure that's right? (S glances at tape-recorder, then loads the first truck, and manages to squeeze seven cars on it). *Seven.* Are you sure that's right now? *No, I'm not sure yet.* (Puts six cars on the next truck). *Six, but there's none on that truck* (pointing to third truck). (E repeats the problem question). *Seven.* Are you sure that's right now? *No, there's seven, and six, and none.* Are you sure that's right? (S nods). Do you have the same number of cars on every truck? *No, but there aren't enough cars; they're all used up.* Why did you put this many on these two trucks? *There aren't enough for nine on every one.* Did you want to fill the truck up? (S nods). Why did you want to do that? *Because then there wouldn't be any more room.* Why didn't you want to leave room? *Er . . .* (E repeats this question several times but gets no response). Do we have enough cars to put nine on every truck, or seven on every truck? (S shakes his head). Did you count the cars to see if there were enough? *No.* Why not? *Er . . . I don't know.*

Phil -- Grade 2: (S points to each car in turn, counting in whispered tones. He then puts six cars on the first truck and five on the second truck, looks at the cars remaining, then sweeps all the cars off the trucks. He then puts one car on each truck, carefully turning each car to face the front, until there are four on each truck, and he holds the thirteenth car in his hand). *Four.* Are you sure that's right? *Yeah.* Quite sure? (S nods). Why didn't you put that one on? *Because you said we have to put the same amount on the three*

trucks. So, do we have to leave that one off? *Yes.* How did you know there would be four on each truck? *Because . . . er . . . because you put each of them one at a time and then we find out we have one left.* Now, when you finished putting one at a time on each truck did you know they had the same number of cars, without counting? *No, but I checked.* But, could your way make sure that all the trucks had the same number of cars? *Yeah.* You started to put six cars on there (indicating first truck), didn't you? *Yeah.* Why did you do that? *I never . . . I never realized that there weren't enough.* You thought there would be enough for six on each truck? *Yeah.* Did you want to fill up each truck? *Uh-huh.* Is that why you put six? *Uh-huh.* Why did you want to fill the trucks? *I thought there would be enough cars.* Does it work that way? *No, because there aren't enough.*

Kathy -- Grade 3: (S looks at the trucks, then loads five cars crosswise on the first truck, thus filling it. She then does the same on the second truck, and puts the remaining three cars on the third truck. She seems to puzzle over this situation, looking from one truck to the next. She points to each individual car on the first two trucks, apparently counting, then looks again at all three trucks. One car is removed from the first truck and put on the third truck. S looks expectantly at E, who repeats the question). *Four.* Are you sure that's right? *Er . . . but that one's got five, that one's got four, and that one's got four* (pointing to the respective trucks). (E repeats the problem question). *Five* (after some hesitation). Are you sure that's right now? *Yeah.* How did you know

there would be five? *But then this one* (pointing to third truck) *will have three*. What made you say five? *I dunno.* Did you try to put five on all of them? (S nods). Why did you want to do that? (S shrugs her shoulders). Couldn't you figure out what to do with that extra one? *No.* Did you want to make sure you put all the cars on? *Unless we go like this* (takes the extra car), *and then take this one off* (removes the car from the second truck). And what would that give you then? *Four each.* Is that right? *Yeah.* What about the extra one? What do we do with that? *I don't know.*

Second Problem Setting

Problem A

Debra -- Grade 1: (S looks at cars, then at trucks). *I think six will be on each truck.* Are you sure that's right? *Yeah.* Quite sure? (She nods). How do you know there will be six? *I dunno; because I can see that this has lots of room* (pointing to the first truck). Do you think there's enough room for six? *Yeah.* Why? *Because . . . er . . . I dunno.* Why do you want to fill each truck up? *Because . . . er . . . I dunno. Because . . . er . . . they take them to where the other cars are.* Do we have enough cars to put six on each truck? (She nods). Did you count them to see if there were enough? *I didn't count them.* But you think there are enough, do you? *Yeah.* I see. Did you think about the men being in the way (pointing to the two men on the trucks)? *No.*

Tammy -- Grade 2: (S looks at cars, pointing and counting sub-vocally). *Four.* Are you sure that's right? *Yeah.* Quite

sure? (She nods). How did you know there would be four? *Because there's thirteen of them* (there are only twelve). Uh-huh, but how do you know there's going to be four on each truck then? *Because I went like that* (indicating groups of four); *there's four, and four, and there's four left.* How did you know to make groups of four? *I saw two there and then two there, and then I put my finger on* (indicating first group of four cars), *for four.* Yes, but why didn't you put your finger on for three? Did you try three? *No.* Did you try five? *No.* You tried four first, did you? (She nods). What if you tried four and it didn't work? *I'd try three or five.* Did you try four because you thought it would be right? *Yeah.* Did you count one, two, three, four, like that? *Yeah.* You show me what you did. *I put my finger on four* (indicating first group of four cars), *then on those four* (pointing to next group of four), *then there was four left, so I had four on each truck.* Did you have a look to see how much room there was on the trucks? *No.* Did you think about the men on the trucks being in the way? *No.* What would you do about the men if you couldn't fit four cars? *I'd take them off.*

Raymond -- Grade 3: (S looks at cars, apparently counting). *Four.* Are you sure that's right? *Yeah.* Quite sure? (He nods). How do you know there's going to be four? *Eight plus four equals, er . . .* How did you know there was going to be four? *I counted them.* You show me what you did. How did you count? *Four here, four here, and four there* (pointing to groups of four). How did you know there would be four in each group? *Because there's*

four here for this truck, four here for this truck, and four here for this truck (indicating matching groups and trucks). But how did you figure out there would be four in each group? *I counted them with my eyes.* You just looked, did you? Did you try making any other groups? *Yeah, I tried two, and three, and four.* You tried with two, did you? *I started with two.* Did three work? *There was three on here, three on there, three on there* (indicating trucks), *but there'd be three left over.* What did you try next? *Four.* What about the men, did you take any notice of them? *No, I've got men just like that. I've got a whole bunch of them at home.* And did you think about how much room there was on the back of the trucks? *Yeah.* Could you fit more than four cars? *Yeah.* Wouldn't you put more than four cars? *No, because there isn't enough cars.*

Problem B

Chistopher -- Grade 1: (S looks at cars, then at trucks). *Four.* Are you sure that's right? *Yeah.* Quite sure? *Yeah.* How did you know there would be four? *Because I think there's enough room for four.* I see. Do you think we could fit more than four on each? *I think we could fit five.* You think we could put five, do you? *Yeah.* Could we put six on each? (He shakes his head). I see. *But I think we could get six on this one* (pointing to the third truck with no man on back). Why not six on these two at the front? *I dunno, because they're smaller than this one.* Why are they smaller? *I dunno.* Is that because there is a man on each of those, is it? *Yeah.* Couldn't the men come off? (He shakes his head). So you would put five on each.

What do you think the answer is then? *Five*. Did you count the cars to see if there were enough to put five on each? *One, two, three,* . . . (pointing at cars). Did you count them before? (He shakes his head and proceeds to point and count the thirteen cars out loud). Do you think that's enough to put five on each? *Yeah*. Why wouldn't you take the men off? *I dunno*.

Michael -- Grade 2: (S glances at the cars, then at the trucks. He points to spaces all over the back of the first truck, counting sub-vocally, then does the same on the second truck). *Twelve*. Do you mean there will be twelve on each truck; twelve on that one, twelve on that one, and twelve on that one (pointing to each truck in turn)? *Yeah*. Are you sure that's right? (He pushes three cars up the ramp onto the first truck, then lifts two more on. He goes to put a sixth car on but hesitates, then eventually squeezes this on without taking the man off. He puts six cars on the second truck, carefully lifting the man and standing him on top of a car. He puts the remaining car on the third truck, and looks expectantly at E. E repeats the problem question. S points to and apparently counts the cars on each truck). *Six*. Are you sure that's right? *I guess so*. Are there enough cars to put six on every truck? *No*. Why did you want to put six? *I don't know*. Why didn't you take the men off the trucks? *I didn't think I was allowed to*.

Harry -- Grade 3: Do you understand? *Yeah, but I don't know how many cars will be on each truck*. Well, that's what you have to figure out, okay? *Yeah, so* . . . (He looks at the trucks and pushes a car up the ramp onto the first truck, and lifts four more

cars on). *Can you move the man?* (S carries on with the task, so E does not respond. S moves this truck forward). *That one over there* (then he moves the second truck up to the ramp and puts five cars on this, then moves all three trucks forward so that the third one is by the ramp). *Er . . . looks like we run out of cars.* He puts the remaining three cars on the third truck). *We've run out of cars.* (E repeats the problem question). *Er . . . five.* Are you sure that's right? *Yeah.* Quite sure? *Yeah.* Why do you say there's going to be five? *'Cause look it. Here's five numbers* (pointing to first truck), *here's another five* (second truck), *there's three, only* (third truck); *so, if you put five cars on each truck, and you only have three more left means you can only put three on one truck.* Uh-huh, but do we have the same number on every truck? *No.* Well, why did you leave them like that then? *We've got too many cars. You see, if I took this away* (moves one car from the second to the third truck) *and put it back here, we still got one car* (pointing to first truck with extra car). *So, we gotta put this one somewhere.* Uh-huh. Did you check to see how many cars there were? *Yeah, there's thirteen.* Why did you put five on those first two trucks? Why didn't you put six, or four, or some other number? *Because then I wouldn't have enough to put here* (third truck). But did you know you wouldn't have enough to put there when you started? *Yeah, but . . .* Why did you put five on the first truck? *Because the man was in the way.*

Third Problem Setting

Problem A

Cameron -- Grade 1: (S glances at tape-recorder, then looks at cars and trucks intently. He puts three red cars on the red truck, three yellow cars on the yellow truck, and three blue cars on the blue truck). *Three.* Are you sure that's right? *Yes.* Quite sure? *Yes.* How did you know there was going to be three? *Because there's no more room.* No more room there? (E points to empty space on a truck). *Yes.* What do you mean then? (No response). Why did you put the blue cars on the blue truck, and the yellow cars on the yellow truck, and the red cars on the red truck? Why did you do it that way? *Because there was no more blue ones.* I see. *And there would be more of these* (indicating yellow and red cars left over) *than the blue ones.* So you could only put three on those trucks as well. *Yeah.* Why did you want to put the blue cars on the blue truck . . . (repeating as before)? *Because they match colors.* Why did you want to match the colors? *Because they look nice like that.* Is that the only reason? *Yeah.*

Lawrence -- Grade 2: (S looks at the trucks, then the cars. He puts a red car on the red truck, a yellow car on the yellow truck, and a blue car on the blue truck; then continues to repeat this process until he gets to the last car). *There's no more blue ones* (as he hesitates then places the remaining red car on the blue truck). (E repeats the problem question). *Four.* Are you sure that's right? *Yeah.* Quite sure? *Yeah, I'm sure.* How did you know there

would be four on each truck? *'Cause I can easily see. 'Cause two go here and two go here* (pointing to cars on truck). Uh-huh. Did you count to see if there were four on every truck when you finished; or did you only count them on one truck? *I counted while I was going along.* I see. How did you count? *I went one, one, one* (pointing to each truck in turn), *two, two, two, . . .* I see. Why did you put the blue cars on the blue truck, and the yellow cars on the yellow truck, and the red cars on the red truck? *I felt like it.* Why did you feel like it? *'Cause they're the same color as the truck.* Why did you want them the same? *'Cause so every truck can carry the same color of cars.* But, you've got a red one on there (blue truck). Does that matter? *No, it doesn't matter.* But you wanted to put the same colors together, did you? *Yeah.* Why did you want to do that? *I just felt like doing it.*

Karen -- Grade 3: (S looks at cars, apparently counting). *Nine will . . . twelve and nine is . . . nine is three, so it's either nine or three on each truck.* (E repeats the problem question. S looks at cars for a moment). *If we put four on each truck there will be thirteen.* Is that thirteen altogether, do you mean? *Thirteen altogether, yeah.* How many will be on each truck? *Four will be on each truck.* Are you sure that's right, now? *Yeah.* Quite sure? *Yeah.* How do you know there's going to be four? *Because we divide these four and put them on this truck, and then put these four on this truck, and then put these four on that truck* (indicating the first four cars for the first truck, etc.). Uh-huh, and will that give you the same number on each truck? *Uh-huh.* How did you figure

out there was going to be four on each truck? *Because I added these with four, so that will make eight, and that four is thirteen* (again indicating groups of four). Did you count all the cars to see there was thirteen? *There's twelve. But you can divide it into thirteen -- four, four, and four* (indicating groups again). Before, you said nine or three. What did you mean? *Because nine plus three* (indicating blue cars) *equals twelve, and one would have less than the others.* I see. Did you think about putting the red ones on the red truck, the yellow ones on the yellow truck, and the blue ones on the blue truck, or not? *No, because if you put . . .* (E interjects). Did you think about that before I mentioned it? *Yeah, I thought about it, but then there'd be only three blues on this truck, and then four yellows on this, and then five reds on that truck* (pointing to respective matching trucks). Why did you want to do it that way? *Because then they would match these three trucks.* I see, but why do you want the colors to match? *Er . . . I dunno.* Did you want them to match, or did you think they had to match? *I thought they had to match.* And did you want them to, as well? *No, not really.* Tell me why you thought they should match. What made you think that? *Er . . . I'm not sure.*

Problem B

Lance -- Grade 1: (S looks at cars, then at trucks). *Six.* Are you sure that's right? (S nods). Quite sure? *There'll be six, and three, and five.* There'll be six, and three, and five, will there? *Yeah.* Are you sure that's right now? *Uh-huh.* How

did you get six, and three, and five? *Simple*! How? *I just guessed.* How did you guess? *By counting.* You show me what you counted. What did you count? *This one, and all these* (pointing to red cars). You counted all those cars did you? How many were there -- do you remember? *I never got to count all those* (apparently considering all the cars). How did you know there was going to be six, and three, and five then? *Count them. I . . . I'm gonna count them.* You show me the six you counted. *This one* (pointing to each red car), *like one, two, three, four, five, six.* I see, and where would you put those? (he points to the red truck). What would you put on here (the yellow truck)? *The yellow cars.* Uh-huh. Why would you put the red cars on the red truck and the yellow cars on the yellow truck? *Because they match.* Why do you want them to match? *I dunno, I like to see both colors the same.* Why do you like the colors to be the same? *'Cause I like those colors to be the same.* Will that give us the same number of cars on every truck? *No.* Well, why didn't you put the same number of cars on each truck then? *Because there wasn't that number. There wasn't . . . there wasn't six of all, or five of all, or three of all.* Why didn't you mix up the colors to put the same number on each? *I dunno.*

Sherry -- Grade 2: (S looks at trucks). *Er . . . eight.* Are you sure that's right? (She nods). Quite sure? *Huh?* Quite sure that's right, are you? (She nods again). How did you know there's going to be eight? *Because the thing on the truck looks like there can be eight cars on it.* I see, uh-huh. And what about the cars? Do we have enough cars to put eight on each truck? (She

looks at the cars and shakes her head). Well, why did you say there's going to be eight on each truck if we don't have enough cars for that? *I don't know.* Did you think there would be enough? (She nods). Why didn't you check? (She shrugs). Did you think about putting the red cars on the red truck, and the yellow cars on the yellow truck, and the blue cars on the blue truck? (She shakes her head). Would that be a way of doing it, or not? (Shakes her head and looks at cars). Why not? *Yeah.* Would that give you the same number of cars on each truck? *No.* Would you do it that way? (Shakes her head).

Kelly -- Grade 3: (S looks at trucks, then at cars, apparently counting). *Five.* Are you sure that's right? (She counts two groups of five). *Wait . . . no.* No? *Uh-huh.* (E repeats the problem question). *Mmm . . .* (She looks at cars again, pointing and apparently counting; is pensive for some time). *Seven?* Are you sure that's right? *Er . . . I'll check.* (She points and checks for two groups of seven, then checks cars again, apparently puzzled). *No, they can't, because there's not even two sets of seven.* (E repeats the problem question. She looks at cars again). *Er . . . this is hard. Does it have to be the same amount of numbers?* (E repeats the question). *Mmm . . .* (as she looks at cars). *We'll have to leave one out. Can we leave one out?* (E repeats question again. S appears to count cars by twos). *Two?* Are you sure that's right, now? (She points and counts cars several times). *Four.* Are you sure that's right, now? *Yeah, but you have to . . . if you leave this one out* (indicating last car in the row). Quite

sure? (S checks for groups of four). *Uh-huh.* Why did you say five before? *I dunno.* Did you think about putting the red cars on the red truck, and the yellow cars on the yellow truck (S interjects: *Uh-huh*), and the blue cars on the blue truck? *Uh-huh.* Did you think about that before I told you? *Uh-huh.* Why would you do that? Did you want the colors to match? *Yeah, sorta. I thought it would be like that.* And would it work that way? *No.*

Fourth Problem Setting

Problem A

Lonnie -- Grade 1: (S glances at tape-recorder, then at trucks. He puts six cars on the first truck by fitting each one in a marked bay, then does the same on the second truck. He looks at E, who repeats the question). *Six. There's six on this one and six on that one.* Are you sure six is right? *Uh-huh.* Quite sure? *Uh-huh.* How did you know there were six? *Because, see there's one, two, three, four, five, six* (pointing to cars), *and there's one, two, three, four, five, six* (on second truck). What about this truck here (pointing to empty truck)? How come you haven't got any on that? *Because there's not enough . . . because six and six is more than none.* I see; but didn't you have to put the same number of cars on every truck? *Mmm, but there wasn't enough for this one.* Why did you fill those trucks up? *Because they gotta go somewhere.* But what about putting the same number on every truck? Didn't you have to do that too? *If I put three here and three here* (the empty truck), *then there'll be six on both*

of these and six here. Do you need six more to put the same number of cars on every truck? (He nods). Can't you put the same number on every truck with those cars? *I could put three on here* (moving three cars from the second truck to the empty one) *like so, there'll be one, two, three, four, five, six* (the three cars and three spaces on the second truck); *one, two, three, four, five, six* (similarly on the third truck). Is that all you can do, is it? *Uh-huh.*

Erin -- Grade 2: *I know; put them in the squares* (and she puts six cars in the bays on each of the first two trucks). *Six.* Are you sure that's right? *One of these don't have one. One's missing out* (the empty truck, apparently). (E repeats the problem question). *You mean altogether?* (E repeats question again). *Six.* Are you sure that's right, now? *Uh-huh.* Quite sure? *Yeah.* Now, how do you know there will be six? *By looking at them.* Did you count them as you put them on? *No, I put 'em on then I counted. See, I went two, four, six* (indicating pairs on first truck). I see. Have you got the same number of cars on every truck? *No, this one* (the empty truck) *hasn't got any.* But didn't you have to put the same number of cars on every truck? *Yes, but there wasn't enough cars.* You didn't have to fill the trucks up, did you? *Yeah, but there wasn't enough cars.* Did I tell you to fill the trucks? *No.* I said to put the same number on every truck. *Yeah, but there wasn't enough cars.* But couldn't you put some cars on that truck (the empty one) so they would all have the same number of cars? *But then they wouldn't all have six.* Do they all have to have six? *No.* Well, do you think that you could put some on that truck so they all have the same number? *Uh-huh.* (She quickly

takes two cars from each of the full trucks and puts them on the empty one). *There, they all have four now.* Why didn't you do that before? *I wasn't thinking about that.* What were you thinking about? *I was thinking of putting them all six.*

Jenny -- Grade 3: (S looks at the cars and the trucks, then puts three cars on each truck in turn, followed by one car on each truck). *Done.* (E repeats the problem question). *Four.* Are you sure that's right? (She counts cars on each truck and nods). Quite sure? *Uh-huh.* How did you know that there was going to be four? *Because there's not enough . . . there's not enough for each, to fill the squares on the back. There's not enough for each car . . . er . . . truck.* Not enough to fill each truck, do you mean? *Yeah.* Why didn't you start to fill up the trucks? *It wouldn't work, because I tried it.* I see. Why did you put three on to start with? *To see if it would work, and see how many's left over.* Why did you think about filling all the spaces? *Er . . . to check if it would work.* Would it? *There wasn't enough cars.*

Problem B

Jennifer -- Grade 1: (S puts one car in each marked bay on the first truck, counting out loud to six as she goes along, and then does the same for the second truck). *There's only one left* (as she puts the remaining car on the third truck). (E repeats the problem question. S points and counts all the bays on the three trucks). *Seventeen. I counted them.* How many will be on each truck? *Six.* Are you sure that's right, now? *Yeah.* Quite sure? (She nods). How did you know there would be six on each truck?

Because I counted when I put the cars on. Have we enough cars for six on every truck? *No.* Did you count before to see if there were enough? (Shakes her head). Did you think there were enough? *I didn't think there were.* What did you think? *I think . . . when . . . when I saw them I didn't think there was enough for three trucks; only for two.* Well, why didn't you put less than six on then? Why didn't you do that? *Because I filled up every square.* Why did you fill up every square? *I filled up every square like that* (pointing to a filled truck).

Jeffrey -- Grade 2: (S looks at cars then at trucks). *Six.* Are you sure that's right? *Er . . . I think so. Yeah, I think so.* Quite sure? *Yeah.* How do you know there will be six? *Because you can tell by the lines you used; two lines down there and one there* (pointing at lines). *There's three in there and three in there* (the bays), *and three plus three is six.* I see. Do we have enough cars to put six on every truck? (He looks at the cars, apparently counting). *No . . . impossible.* So, why did you say six then? *Now I'm sure that there's not gonna be enough, because there's three here and three here* (first truck), *and three here and three here* (second truck), *and there's gonna be one on that truck* (third truck), *and six on there and six on there* (the first two trucks). Why would you do that when there aren't enough for six on every truck? *Er . . . I dunno.*

Caroline -- Grade 3: (S looks at cars, then at trucks. She puts six cars on the first truck, six cars on the second truck, then the remaining car on the third truck). *Thirteen altogether.*

(E repeats the problem question). *Six, six, one* (pointing to each truck in turn). Are you sure that's right? (She nods). Quite sure? *Huh?* Are you quite sure that's right now? (She nods). Why did you put six on? *Because there's six spaces for each of them* (indicating the trucks). Do you have the same number of cars on every truck? *No.* Why didn't you put the same number on every truck? *Because there's not enough for all the trucks.* But, if you put less than six, wouldn't there be enough? (She nods). Why didn't you put less than six, then? *I dunno.*

Fifth Problem Setting

Problem A

Dawn -- Grade 1: (S pushes the first five cars up the ramp onto the blue truck putting one car in each space, moves this truck forward and drives the yellow truck to the ramp, then loads that truck in the same way. She moves both of these trucks forward and drives the red car up to the ramp, then puts the remaining two cars on it). *There's two gonna be on this one.* (E repeats the problem question). *Five.* Are you sure that's right, now? *But there's supposed to be a man right there* (pointing to right front bay on red truck where there is no man, corresponding to position of man on each of the other trucks). *So there will be six there* (red truck). (E repeats the problem question again). *Five.* Are you sure that's right? *Uh-huh.* Quite sure? *But there's six . . . but there's supposed to be six there* (red truck). Why did you put

five on these? *Because the spaces; but there's two on that one, but there's supposed to be six.* We had to put the same number on every truck. Do we have the same number on every truck? (Shakes her head). Why didn't you put the same number on every truck? *Because there's just two more left.* Uh-huh. Couldn't you put less than five? *No.* Why not? *Because the man was there.* Couldn't you put less than five? What about four on each -- would that work? *No.* No? *No, because there'd be a few more spaces left.*

Glen -- Grade 2: (S points to, and apparently counts the empty bays on each truck). *There'll be five on these two trucks* (the blue and yellow), *and six on this one* (the red truck). (E repeats the problem question). *Six on each truck.* Are you sure that's right? *You mean if you put all the trucks . . . all the same amount on every truck?* (E repeats the question again). *There'll be six.* Are you sure that's right, now? *Yeah.* Quite sure? (He nods). How did you know there was going to be six? *Well, because there's six squares.* I see. Why did you say five before, on those two trucks? *Because there's them two men there.* Can't the men move off? *Yeah.* Did you think about putting the blue cars on the blue truck, the yellow cars on the yellow truck, and the red cars on the red truck? Did you think about that? *No.* Have we enough cars to put six on every truck? (He counts the cars). *No.* Did you count the cars before? *I didn't count the cars. I counted the spaces.* Do you think it would have helped if you counted the cars? *I dunno.*

Curtis -- Grade 3: (S looks at trucks and cars, glances at E, then looks at cars again, apparently counting). *Four.* Are you sure that's right? *Yeah* (after some hesitation). Quite sure? (He nods). How do you know four is right? *I just counted the cars.* You show me how you counted. *Er . . . like . . . first I did umm . . . six, but that wasn't right, and so I did four.* I see. What made you try six? *Er . . . because there's six squares on each truck.* What about the men being on the trucks? Does that make any difference? *Er . . .* Did you think about the men being in the way? (He shakes his head). And did you think about putting the blue cars on the blue truck, the yellow cars on the yellow truck, and the red cars on the red truck? *Uh-huh.* Did you think about that before I told you? *Yeah.* What made you think about that? *Er . . . I dunno.* Why didn't you go on and do it that way? *Because there's only three blue cars.* How do you know four is right? *Because that's the same amount on every truck.*

Problem B

Michael -- Grade 1: (S glances at cars, then at trucks). *Six.* Are you sure that's right? *Yeah, because they're so small you could just fit six in.* Uh-huh. *Even though there's not much room between the lines you can still fit six in.* So you would put six on there, six on there, and six on there (pointing to trucks). Is that what you would do? *Yeah.* And what about the men being there? What do we do with them? *Put them in the truck.* Do we have enough cars to put six on each truck? *Yeah.* Did you count them to see if there

were enough? *No.* How do you know there are enough cars to put six on each truck? *Because three and three is six* (pointing to bays on yellow truck). But how do you know there are enough cars to put six on every truck? *Oh, I just know.* Did you think about putting one in every space, did you? *Yeah.* Why would you fill the trucks up? *'Cause I'd try to get as much as I could out.* Why would you do that? *So they could sell them.* I see. Did you think about putting the blue cars on the blue truck, the yellow cars on the yellow truck, and the red cars on the red truck? *Yeah.* Did you think about doing it that way before I told you? *Yeah.* Why would you do it that way? *Because they match.* Why do you want them to match? *'Cause they'd probably look better.* They would look better, would they? *Yeah. This is what kind we have -- a Buick* (pointing to a car).

Anne -- Grade 2: *Yeah, but I don't know how many trucks. I could fill the lot in.* (E repeats the question). *We'll have to squash 'em in there* (as she puts the first five cars on the blue truck). (She puts the next five cars on the yellow truck, then the three remaining cars on the red truck). *I don't get this; there's three left. I'm gonna try another number then.* (She shifts one car from the blue truck onto the red truck, then removes a car from the yellow truck). *There; one car left* (and she waits expectantly). (E repeats the problem question). *I think four.* Are you sure that's right? *I dunno . . . Yeah, because if we put five on each truck there's three left.* Uh-huh. *Now there's one left, and if we put it on either one truck it's uneven.* Are you sure four is right, then? (She nods). How do you know there's going to be four? *Because, if I put five here, five there, there might*

be . . . there were three left then. So I have to take one from there and put it on there, and take one off there (pointing to trucks to indicate actions carried out previously). Uh-huh. What about the colors? Did you think about putting the yellow cars on the yellow truck, and the blue cars on the blue truck, and . . . *I don't think so.* Do you think that matters? *No, because there . . . because there are four yellows, and there are more reds than blues. There's only three blues.* So you don't think that matters, eh? *No, I don't think it'd matter.* Wouldn't it help if you did that? *No, because we have to put the same amount on every truck.*

Celina -- Grade 3: (S glances at cars, then points to and counts spaces on each truck). *Five.* Are you sure that's right? (She looks at the trucks). *There'll be six if you take these men off.* Are you sure that's right now? *Yeah, six.* Why did you say five before? *Because I didn't count the men, and the men stayed there.* Would you take the men off? *Yeah.* Okay. Do we have enough cars to put six on every truck? *Er . . . yeah, I think so.* Did you count to see if there were enough? (She counts the cars to herself). *No.* Didn't you count them before? (She shakes her head). Why didn't you? (No response). Did you think there would be enough for six on every truck? (She nods). I see. Now, did you think about putting the blue cars on the blue truck, the yellow cars on the yellow truck, and the red cars on the red truck? Did you think about that before I told you? *No.*

APPENDIX B

DATA CODING SHEET FORMAT

Subject I. D.	255	266	66	315	108
Sex	M	F	M	M	F
Grade	2	1	3	1	2
Ability Percentile Rank	87	43	13	43	11
Problem Difficulty	B	A	A	B	B
Problem Setting	1	2	3	4	5
Subject Very Nervous=VN;Nervous=N;Relaxed=R	R	R	R	R	N
Number of Looks at Tape-recorder	0	0	1	0	0
Verbal (V) or Manipulative (M) Solution	M	V	M	V	V
Time to Nearest Second for 1st Solution	115	32	218	35	8
Groupings Attempted	6,5,0 4,4,4	3,3,3	3,4,5 4,4,4	6,6,6	5,5,5
First Solution	4	3	3*	6	5
Confirmed Solution	4	3	4	6	5
Score	3	0	2	0	0
Used × or ÷ Process					
Loaded 1 Car per Truck in Rotation	4,4,4				
Grouping Based on Trial(s) of Equiv. Sets			4,4,4		
Solution a Guess		✓			
No Explanation for Solution					
Solution of '3' Based on 3 Trucks					
Distracted by Spatial-Numerical Cue: A=Actual; I=Induced	6,5,0 A			A	A
Overcame Above Distraction	✓				
Reason Given for Above Distraction: N=None; E=Enough Cars;H=Had to;W=Wanted to;O=Other	E			E*	W
Distracted by Color-Attribute Cue: A=Actual; I=Induced			3,4,5 A		I
Overcame Above Distraction			✓		
Reason Given for Above Distraction: N=None; A=Aesthetics;H=Had to;W=Wanted to			W		A
Distracted by Men; A=Actual; I=Induced					A
Overcame Above Distraction					
Reason Given for Above Distraction: N=None; L=Help Load Cars;H=Had to Stay on Trucks; W=Wanted Men to Stay					N
Number of Times Loading Ramp Used					
Engaged in Unnecessary Simulations	✓*				
Counted All Cars During Solving Process	✓				
Solved Problem During Interview					
Relevant Comments	*Turned each car to face front		*Based on 3 blue cars	*Insists there's 18 cars	

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